135. *Abstract Vanishing Cycle Theory*[*]

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1. *Introduction.* In this short note we shall discuss a simplified version of our abstract vanishing cycle theory[1] including the unequal-characteristic case. This theory provides, roughly speaking, abstract analogues of parabolic substitutions which the solutions of differential equations of Picard-Fuchs type undergo around the simplest type of singular points and it can be applied to construct an algebraic theory of modular functions with levels for all characteristics.[2] This we shall discuss separately[3] in the case of elliptic modular functions.

2. *Starting point.* Suppose that R is a discrete valuation ring. In order to be able to apply Hensel's lemma[4] we shall assume that R is complete. Let K be the quotient field and k the residue field. We fix a natural homomorphism of R to k and call its extensions specializations at the center of R.[5] Let C be a non-singular curve defined over K and let C' be its specialization at the center of R. We shall assume that C' is absolutely irreducible. We shall also assume that C' has at most one singularity and that the singularity is an ordinary double point. We note that ordinary singular points are, in a sense which can be made precise easily, generic singularities. At any rate, we shall denote this possible singular point by Q. If g is the genus of C, the genus of C' is either g or $g-1$ according as Q is absent or not. Pick a divisor $\mathfrak{r}$ of C of degree d greater than $2g-2$ rational over K such that the specialization $\mathfrak{r}'$ at the center of R is free from Q. This is always possible and, in fact, we can even assume that $\mathfrak{r}$ is positive. Let J be the Jacobian variety of C constructed by Chow's method[6] with reference to $\mathfrak{r}$. Then the specialization J' of J at the center of R is either the Jacobian variety of C' constructed by Chow's method or a completion of the Rosenlicht variety $(J')_0$ of C' constructed by Chow's method[7] with reference to $\mathfrak{r}'$. Moreover, the image points of $\mathfrak{r}$ and $\mathfrak{r}'$ being taken as neutral elements of J and $(J')_0$, the group law of J is specialized to the group law of $(J')_0$ at the center of R. We proved this *compatibility* only in the geometric case.[8] However the proof can be taken over verbatim to the present case. We also note that the Rosenlicht variety $(J')_0$ is a commutative group variety which contains the group variety G_m of

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