3. Note on Finite Simple c-Indecomposable Semigroups

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In this note we shall report the result of study of finite simple c-indecomposable semigroups except groups without proof, which we shall discuss precisely in another paper. A semigroup is said to be *c-indecomposable* if it has no commutative homomorphic image except one-element semigroup.

1. Finite simple semigroups. A simple semigroup is defined as a semigroup which has no proper ideal.[1]

Referring Theorem 8 in [1],[2] we have

Lemma 1. *A finite simple semigroup without zero belongs to one of the following three categories.*

(1) *Finite simple c-indecomposable semigroups without zero except groups.*

(2) *Finite groups.*

(3) *Finite simple non-commutative non-unipotent semigroups whose greatest c-homomorphic images are non-trivial groups.*

Lemma 2. *A finite simple semigroup with zero belongs to one of the following three categories.*

(1) *Finite simple c-indecomposable semigroups with zero.*

(2) *A z-semigroup of order* 2.

(3) $S=\{0\}\cup S'$ *where* 0 *is a zero of* S, *and* S' *is a finite simple semigroup without zero. We permit* S' *to be a one-element semigroup.*

As a special case, we get

Lemma 3. S *is a finite commutative simple semigroup without zero if and only if* S *is a finite commutative group.* S *is a finite commutative simple semigroup with zero if and only if* S *is either a z-semigroup of order* 2 *or a finite commutative group with zero adjoined.*

2. Finite simple c-indecomposable semigroups with zero. According to Rees [3], a finite simple semigroup S is completely simple, and hence it is faithfully represented as a regular matrix semigroup over a group. The defining matrix $P=(p_{\mu\lambda})$ of S is said to contain a zero if there is an element $p_{\beta\alpha}=0$ at least.

Without the condition of finiteness, we have

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1) By a proper ideal T of a semigroup S we mean a proper subset T of S such that $T\neq\{0\}$, $ST\subseteq T\neq S$, and $TS\subseteq T\neq S$.

2) Numbers in brackets [ ] refer to the references at the end of the paper.