17. Convergence Concepts in Semi-ordered Linear Spaces. II

By Hidegorô NAKANO
Hokkaido University

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In the part I*) we discussed the standard modificators in the case where R is super-universally continuous, and we obtained Theorems 3 and 4. In the sequel, these theorems will be extended to more general cases which are essentially important in the theory of semi-ordered linear spaces.

An operator $\mathfrak{a}$ is said to be *reducible*, if $(Pa_\nu)^{\mathfrak{a}} = Pa_\nu^{\mathfrak{a}}$ $(\nu=0, 1, 2, \cdots)$ for every projection operator P on R. A modificator A is said to be *reducible*, if every operator of A is reducible. All sub., loc. and ind. operators are obviously reducible, and hence S, L, I and all standard modificators are reducible. We see easily that AB and $A\circ B$ are reducible, if both A and B are reducible. Every reducible modificator commutes evidently all loc. operators by definition.

A semi-ordered linear space R is said to be *locally super-universally continuous*, if R is continuous and we can find a system of projectors $[p_\lambda]$ $(\lambda \in \Lambda)$ such that $\bigcup_{\lambda\in\Lambda}[p_\lambda]=1$ and $[p_\lambda]R$ is super-universally continuous for all $\lambda\in\Lambda$.

Lemma 5. *If R is locally super-universally continuous, then we have*

$$ALSB \succ LASLB$$

for every two reducible modificators A and B.

Proof. Let $[p_\lambda]$ $(\lambda\in\Lambda)$ be a system of projectors such that $\bigcup_{\lambda\in\Lambda}[p_\lambda]=1$ and all $[p_\lambda]R$ $(\lambda\in\Lambda)$ are super-universally continuous. Recalling Lemma 4, we have $ALSB\succ ASLB$ in $[p_\lambda]R$ for every $\lambda\in\Lambda$. Thus we have in R

$$ALSB \succ LALSB \succ LASLB.$$

Lemma 6. *If R is locally super-universally continuous, then*

$$(L\circ S)(L\circ S) \sim SLS.$$

Proof. As $L\circ S \geqq LS$ by (2), we have by (3)

$$(L\circ S)(L\circ S) \geqq (L\circ S)LS.$$

We suppose $a_0 = (L\circ S)LS\text{-}\lim_{\nu\to\infty} a_\nu$. Then, by virture of Theorem 1, we can find $\mathfrak{L}_0\in L$ and $\mathfrak{S}_0\in S$ such that

$$a_0^{\mathfrak{l}\mathfrak{s}} = LS\text{-}\lim_{\nu\to\infty} a_\nu^{\mathfrak{l}\mathfrak{s}} \quad \text{for all } \mathfrak{l}\in\mathfrak{L}_0,\ \mathfrak{s}\in\mathfrak{S}_0.$$

As R is locally super-universally continuous, we can suppose here that

*) H. Nakano and M. Sasaki: Convergence concepts in semi-ordered linear spaces. I, Proc. Japan Acad., **35**, no. 1 (1959).