

27. On an Extension Theorem

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In an earlier paper^{*)} I stated an Extension Theorem. Recently T. Itô found out a counter-example to it. Here I want to correct it as follows.

Extension Theorem. *Let R be a linear space and m a functional on R subject to*

- 1) $-\infty < m(x) \leq +\infty$ for every $x \in R$,
- 2) $m(\lambda x + \mu y) \leq \lambda m(x) + \mu m(y)$ for $\lambda + \mu = 1$; $\lambda, \mu \geq 0$,
- 3) $m(\xi x) = +\infty$ for all $\xi < 0$, if it holds for all $\xi > 0$.

For a linear manifold A of R and a linear functional φ on A , if

$$\varphi(x) \leq m(x) \quad \text{for every } x \in A,$$

then we can find a linear functional ψ on R such that

$$\psi(x) = \varphi(x) \quad \text{for every } x \in A,$$

$$\psi(x) \leq m(x) \quad \text{for every } x \in R.$$

^{*)} H. Nakano: An Extension Theorem, Proc. Japan Acad., **33**, 603-604 (1957).