

23. Supplement to "Homomorphisms of a Left Simple Semigroup onto a Group"

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In this supplementary note of [1], we show that Theorem 3 in [1] holds even when we omit the condition that ' N contains the core of S ' in its assumption, and that, indeed, any normal and left unitary subsemigroup of a left simple semigroup S contains the core of S .

Let N be a normal and left unitary subsemigroup of a left simple semigroup S . Then, *without the assumption that N contains the core of S* , from Lemma 6 onwards in [1], we can argue in the same way as in [1], except the parts which we shall discuss in the following.

At first, we show that Lemma 7, asserting that $a \in Na$ for any $a \in S$, holds in this case. In fact, we take an element $x \in N$. By the left simplicity of S , there exist two elements y and n of S such that $yx = a$ and $nx = x$. By Lemma 6, we have $n \in N$. Moreover, since N is normal, there exists an element $n' \in N$ such that $yn = n'y$. Then we have

$$a = yx = ynx = n'yx = n'a \in Na.$$

Also, in the proof of Lemma 10 in [1], the statement "Then there exists an element $u \in S$ such that $ua = a$. The element u belongs to U and so belongs to N " is replaced in this case by the statement "Then by Lemma 7, there exists an element $u \in U$ such that $a = ua$ ".

After discussing in the same way as in [1] except the parts above-mentioned, we obtain Theorem 3 in [1] in a better form:

Theorem 3'. *If N is a normal and left unitary subsemigroup of S , then there exists a homomorphism θ of S onto a group such that the kernel of θ is N .*

As an immediate consequence of the above theorem, we obtain the following

Theorem 4. *If N is a normal and left unitary subsemigroup of S , then N contains the core of S .*

In fact, by Theorem 3', there exists a homomorphism θ of S onto a group G such that the kernel of θ is N . But by Theorem 2 of [1], N , being the kernel of θ , contains the core of S .

Reference

- [1] T. Saito: Homomorphisms of a left simple semigroup onto a group, Proc. Japan Acad., **34**, 664-667 (1958).