35. On the Capacitability of Analytic Sets

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1. The results, obtained by Choquet [2], on the capacitability were extended by Aronszajn-Smith [1] and the author [3] as follows. Every analytic set in the τ-dimensional Euclidean space is capacitable with respect to the capacity of order α, where $0<\alpha<\tau$ [1]. Let, in general, Ω be a locally compact space, every compact subset of which is metrisable and suppose that a positive symmetric kernel function satisfies Frostman's maximum principle. Then every analytic set in Ω, which is contained in a compact set, is capacitable with respect to the capacity defined by admissible measures [3].

This note will communicate some extensions of these results, details of which will be published later.

2. Let Ω be a locally compact separable metric space, and let Φ be a positive symmetric kernel function which satisfies the following two conditions:

1° the continuity principle, that is, the continuity of the restriction of any potential U^{μ} of a positive measure μ to its carrier S_{μ} implies the continuity of U^{μ} in Ω,

2° when Ω is non-compact, there exists, for any compact subset K and for any positive number ε, a compact subset $L\supset K$ such that $\Phi(P,Q)<\varepsilon$ in $K\times(\Omega-L)$.

Since Ω is separable, there exists an exhaustion $\{\Omega^{(m)}\}$ $(m=1,2,\cdots)$ of Ω such that each $\Omega^{(m)}$ is an open set whose closure is compact, $\Omega^{(m)}\subset\Omega^{(m+1)}$ and $\Omega=\bigcup_{m=1}^{\infty}\Omega^{(m)}$. In the following consideration we take an exhaustion $\{\Omega^{(m)}\}$ of Ω and we fix it. We say that a sequence $\{\mu_n\}$ $(n=1,2,\cdots)$ of positive measures converges vaguely to a positive measure μ when it has the following properties:

(1) it converges vaguely to μ in the ordinary sense,

(2) for each m, the sequence $\{\mu_n^{(m)}\}$ of the restrictions of μ_n to $\Omega^{(m)}$ converges vaguely in the ordinary sense to $\mu^{(m)}$ which coincides with μ in $\Omega^{(m)}$

3. Now let μ be a positive measure whose total measure is finite. Every subset of a set, at which U^{μ} is infinite, is called a polar set. We denote by $\mathfrak{P}$ the family of all polar sets. Obviously $E=\bigcup_{n=1}^{\infty}E_n$ is a polar set, if every E_n is a polar set.

For an arbitrary set X we consider the following families $\mathfrak{F}_X$ and