62. *Embeddings of Projective Spaces into Elliptic Projective Lie Groups*

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The real, complex and quaternion projective spaces are topologically closely connected with the classical Lie groups (orthogonal group $O(n)$, unitary group $U(n)$ and symplectic group $Sp(n)$). For example, the projective spaces can be embedded into the classical Lie groups. This inclusion map φ is defined by

$$\varphi([x_1, x_2, \cdots, x_n]) = (\delta_{ij} - 2x_i\bar{x}_j) \qquad i, j = 1, 2, \cdots, n,$$

where $|x_1|^2 + |x_2|^2 + \cdots + |x_n|^2 = 1$, and φ plays some important role to study the topologies of the classical Lie groups [4]. These embeddings are extendable to the case of the field of octanions (i.e. Cayley numbers). That is, in this paper, we shall show that *the octanion projective plane* Π *can be embedded into the group* F_4 which is a compact simply connected F_4-type exceptional simple Lie group.

1. Let F be the field of real numbers R, complex numbers C, quaternions Q or octernions $\mathfrak{C}$.

Let $\mathfrak{J}$ be the set of all hermitian matrices of 3 order

$$X = \begin{pmatrix} \xi_1 & x_3 & \bar{x}_2 \\ \bar{x}_3 & \xi_2 & x_1 \\ x_2 & \bar{x}_1 & \xi_3 \end{pmatrix}$$

with coefficients in F. We define the Jacobi multiplication in $\mathfrak{J}$ by

$$X \circ Y = 1/2(XY + YX),$$

the inner product in $\mathfrak{J}$ by

$$(X, Y) = tr(X \circ Y),$$

an another multiplication in $\mathfrak{J}$ by

$$X \times Y = 2X \circ Y - tr(X)Y - tr(Y)X + (tr(X)tr(Y) - (X, Y))E^{1)}$$

and define

$$(X, Y, Z) = (X, Y \circ Z).$$

Let $A(\mathfrak{J})$ be the group of all automorphisms of $\mathfrak{J}$, i.e. $\alpha \in A(\mathfrak{J})$ is a non-singular linear transformation of $\mathfrak{J}$ which satisfies

$$\alpha(X \circ Y) = \alpha X \circ \alpha Y.$$

This group $A(\mathfrak{J})$ is characterized that the group of all non-singular linear transformations of $\mathfrak{J}$ which invariant (X, Y) and (X, Y, Z), i.e.

$$(\alpha X, \alpha Y) = (X, Y) \qquad \text{for } X, Y \in \mathfrak{J}$$

$$(\alpha X, \alpha Y, \alpha Z) = (X, Y, Z) \qquad \text{for } X, Y, Z \in \mathfrak{J}.$$

In the case of R (resp. C, Q), for any $\alpha \in A(\mathfrak{J})$, there exists an orthogonal matrix $O \in O(3)$ (resp. unitary matrix $U \in U(3)$, symplectic

1) E is the unit matrix of 3 order.