60. *On the Sets of Regular Measures. I*

By Hitoshi TSUJIMOTO
University of Osaka Prefecture
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1. Introduction. Let (X, S) be a topological measurable space, and let us consider more than one measure on S. The measures of our objects are not necessarily finite. The relations among measurable sets regular with respect to a fixed measure m are well known. Here, the term "regular" is employed as usual: a measurable set E is inner regular with respect to m if

$$m(E)=\sup\{m(C): E\supseteq C, C\in \boldsymbol{C}\},$$

where $\boldsymbol{C}$ is the class of compact measurable sets. The measurable set E is outer regular with respect to m if

$$m(E)=\inf\{m(U): E\subseteq U, U\in \boldsymbol{U}\},$$

where $\boldsymbol{U}$ is the class of open measurable sets. If each measurable set is inner (outer) regular, the measure m will be inner (outer) regular.

About the relations among the regularities of two or more measures, G. Swift [2] investigated chiefly concerning irregular Borel measures, and R. E. Zink [3] concerning integral measures.

We shall now propose and make a study of the following problems:

(1) Let $\{\mu_i\}_{i=1}^{\infty}$ be a sequence of measures and ν be a measure such that $\lim_{i\to\infty}\mu_i(E)=\nu(E)(E\in S)$. Then will the inner (outer) regularities of $\mu_i(i=1, 2, \cdots)$ be preserved on ν?

(2) Let $\mu_1\smile\mu_2(\mu_1\frown\mu_2)$ be the superior (inferior) measure of the two measures μ_1 and μ_2. Then will the inner (outer) regularities of μ_1 and μ_2 be preserved on $\mu_1\smile\mu_2(\mu_1\frown\mu_2)$? Next, does the argument change when we substitute a set of measures $\{\mu_\lambda\}_{\lambda\in\Lambda}$ (of arbitrary numbers) in place of μ_1, μ_2?

(3) Let f_1 and f_2 be two non-negative measurable functions and μ be a measure. Let us define the measures μ_1, μ_2 and ν by means of the equations

$$\mu_1=\int f_1 d\mu, \quad \mu_2=\int f_2 d\mu, \quad \nu=\int\sqrt{f_1 f_2}\, d\mu.$$

Under what conditions will the inner (outer) regularities of μ_1 and μ_2 be induced on ν?

(4) The similar problems with respect to irregular measures

If we deal with finite measures only, the arguments will be very simple, but on the contrary the permission of introducing infinite measures complicates the affairs, because, for instance, $m(E)=\infty$