57. Notes on Uniform Convergence of Trigonometrical Series. II

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1. We consider a series with real terms

$$\sum_{n=1}^{\infty} a_n \qquad (a_0=0),$$

and write

$$s_n^{\gamma}=\sum_{\nu=0}^{n} A_{n-\nu}^{\gamma}\, a_\nu=\sum_{\nu=0}^{n} A_{n-\nu}^{\gamma-1}\, s_\nu \tag{1.1}$$

$$(-\infty<\gamma<\infty),$$

$$t_n^{\gamma}=\sum_{\nu=0}^{n} A_{n-\nu}^{\gamma-1}(\nu a_\nu)=\sum_{\nu=0}^{n} A_{n-\nu}^{\gamma-1}\, t_\nu \tag{1.2}$$

where $s_n=s_n^0$, $t_n=t_n^0$, and $A_n^{\gamma}=\binom{\gamma+n}{n}$. Then, in particular $s_0^{\gamma}=0$, $t_0^{\gamma}=0$, and for $n=1, 2,\cdots$,

$$s_n^{-1}=a_n, \quad s_n^{-2}=a_n-a_{n-1}=-\Delta a_{n-1},$$
$$t_n^0=na_n, \quad t_n^{-1}=na_n-(n-1)a_{n-1}.$$

The object of this paper is to prove some theorems (Theorems 3-5) which will unify the results of Szász [1], Hirokawa [5] and others. This note is a continuation of Yano [6, 7].

THEOREM 1. Let $0<r$, $0<s<1$ (or $s=1, 2,\cdots$) and $0<\alpha\leqq 1$. If

$$\sum_{\nu=1}^{n} |t_\nu^r|=o(n^{1+r\alpha}), \tag{1.3}$$

$$\sum_{\nu=n}^{2n} (|t_\nu^{-s}|-t_\nu^{-s})=O(n^{1-s\alpha}), \tag{1.4}$$

as $n\to\infty$, then the series $\sum a_n \sin nt$ converges uniformly (on the real axis).

THEOREM 2. Under the same assumption as in Theorem 1, the series $\sum a_n \cos nt$ converges uniformly when $0<\alpha<1$, and in the case $\alpha=1$ this series converges uniformly if and only if $\sum a_n$ converges.

These theorems are an alternative form of Theorem 1 in the papers [6] and [7] respectively.

2. THEOREM 3. Let $0<s\leqq 1$, and q be an arbitrary real constant. If

$$(1-x)\sum_{n=1}^{\infty} na_n x^n\to 0 \qquad (x\to 1-0), \tag{A.2}$$

$$\sum_{\nu=n}^{2n} (|\gamma_\nu|-\gamma_\nu)=O(n^{1-s}) \qquad (n\to\infty), \tag{2.1}$$

where

$$\gamma_n=(1+qn^{-1})t_n^{1-s}-t_{n+1}^{1-s} \qquad (n=1, 2,\cdots), \tag{2.2}$$