82. A Remark on the Abstract Analyticity in Time for Solutions of a Parabolic Equation

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1. Consider an equation of evolution

$$du/dt = A(t)u \tag{1.1}$$

where the differential operator

$$A(t) = \sum_{i,j=1}^{m} a^{ij}(t,x)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{m} b^i(t,x)\frac{\partial}{\partial x_i} + c(t,x)$$

is elliptic on a domain G of an m-dimensional Euclidean space. As for the case when all the coefficients of $A(t)$ are independent of t, K. Yosida [7] extended the result of S. Itô and H. Yamabe [5, 6].

In the present note the author will give a direct proof of K. Yosida's result under an assumption that all the coefficients $a^{ij}(t,x)$, $b^i(t,x)$, $c(t,x)$ are uniformly analytic in t for any x in G.

The method is based upon the idea of C. B. Morrey and L. Nierenberg [2]. The result, which is applicable to the unique continuation problem of (1.1) [1], is obviously extended with respect to certain distribution solutions of generalized parabolic equations [4].

2. For the sake of simplicity, we shall discuss the case $G = E^m$ and assume that the real coefficients $a^{ij}(t,x)$, $b^i(t,x)$, and $c(t,x)$ are sufficiently differentiable such that

$$D_x^{(k)}D_t^{(p)}a^{ij}(t,x),\quad D_x^{(k')}D_t^{(p)}b^i(t,x),\quad D_t^{(p)}c(t,x)$$
$$(k=0,1,2;\ k'=0,1;\ p=0,1,2,3,\cdots)$$

are continuous over $[-1,1]\times E^m$, and that there are two positive numbers L and K such that

$$\max_{\substack{k=0,1\\ i,j=1,2,\cdots,m}} \{|D_x^{(k)}a^{ij}(t,x)|,\ |b^i(t,x)|,\ |c(t,x)|\} \leqq L, \tag{2.1}$$

$$\max_{\substack{p=0,1,2,\cdots\\ i,j=1,2,\cdots,m}} \{|D_t^{(p)}a^{ij}(t,x)|,\ |D_t^{(p)}b^i(t,x)|,\ |D_t^{(p)}C(t,x)|\} \leqq Lp!\,K^p \tag{2.2}$$

for any $x\in E^m$, $t\in(-1,1)$. Moreover there are two positive γ and δ such that

$$\gamma \textstyle\sum_{i=1}^m \xi_i^2 \geqq \sum_{i,j=1}^m a^{ij}(t,x)\xi_i\xi_j \geqq \delta \sum_{i=1}^m \xi_i^2 \tag{2.3}$$

for any $x\in E^m$, $t\in(-1,1)$ and for any real $\xi=(\xi_1,\cdots,\xi_m)$. Set

$$\|f(t,x)\|_r^2 = \int_{-r}^{r}\int_{E^m} |f(t,x)|^2\,dx dt$$

and

$$\overline{A(t)} = A(t) - \alpha$$