# 75. On Ring Homomorphisms of a Ring of Continuous Functions. II

By Takesi ISIWATA

Tokyo Gakugei University, Tokyo

(Comm. by K. KUNUGI, M.J.A., July 13, 1959)

Anderson and Blair [1] have investigated representations of certain rings as subalgebras of $C(X)$.[1] In this paper, we shall in §1 also consider such representations of certain rings and we shall improve Theorems 2.2 and 3.2 in [1] using results obtained in [2, 3]. From results in §1, we obtain in §2 new characterizations of locally $Q$-complete spaces, $Q$-spaces, locally compact spaces and compact spaces.

Let $R$ be a ring of all real numbers. A subset $A$ of $C(X)$ is said, according to [1], to be *weakly pseudoregular* if $X$ has a subbase $\mathfrak{U}$ of open sets such that for any $U\in\mathfrak{U}$ and $x\in U$ there are an $\alpha>0$ (in $R$) and an $f$ in $A$ such that $|f(x)-f(y)|>\alpha$ for $y\notin U$. $A$ is *pseudoregular*[2] if for any $x\in X$ and any open neighborhood $U$ of $x$, there is an $f\in A$ such that $f(x)=0$ and $f(y)\geq 1$ for $y\notin U$. An element $f$ in $A$ is said to be *strictly positive* if there exists an $\alpha>0$ (in $R$) such that $f(x)\geq\alpha$ for every $x\in X$. Next suppose that $A$ is an arbitrary algebra over $R$. A maximal ideal $M$ of $A$ is said to be *real* if the residue class algebra $A/M$ is isomorphic to $R$. $\mathfrak{R}_A$ denotes the totality of real maximal ideals of $A$. An element $f$ in $A$ is said to be *strictly positive* if there exists $\alpha>0$ (in $R$) such that $M(f)\geq\alpha$ for every $M\in\mathfrak{R}_A$ where $M(f)=f$ mod. $M$. Let us put $S(f)=\{M(f);M\in\mathfrak{R}_A\}$ which is called a *spectrum* of $f$. If $A$ is a subset of $C(X)$, and for any $M\in\mathfrak{R}_A$, there is a unique point $x$ in $X$ such that $M=M_x=\{f;f(x)=0\}$ then $A$ is said to be *point-determining*; in other words, $A$ has the property $(H^*)$ in [3], that is, any ring homomorphism $\varphi$ of $A$ onto $R$ is a point ring homomorphism $\varphi_x$ and $x\neq y$ implies $\varphi_x\neq\varphi_y$.

**1.** Now suppose that $A$ is a ring such that $\mathfrak{R}_A\neq 0$ and $\bigcap_{M\in\mathfrak{R}_A} M=\theta$ (written $\frown\mathfrak{R}_A=\theta$). We define a function $f^*$ an $\mathfrak{R}_A$ by $f^*(M)=M(f)$, moreover, introduce a weak topology on $\mathfrak{R}_A$, that is, we take as a subbase of open sets of $\mathfrak{R}_A$, $\mathfrak{U}=\{U_M(f,\varepsilon);f\in A,\varepsilon\in R,\varepsilon>0\}$ where $U_M(f,\varepsilon)=\{N;|M(f)-N(f)|<\varepsilon,N\in\mathfrak{R}_A\}$. Then, by [1, Theorem 2.1], for any given $X$, a weakly pseudoregular point-determining subring $A$ of $C(X)$

1) In the following, $X$ is always a completely regular $T_1$-space and other terminologies used here, for instance $C(X)$, ring homomorphisms and local $Q$-completeness, are the same as in [2, 3].

2) The definition of pseudoregular in [1] requires moreover that $A$ contains a constant function $e$ which takes value 1 on $X$.