74. On Compact Semirings

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1. *Introduction.* In this paper we generalize to the infinite case our theorem that *a finite semiring without zeroid is a ring* [1]. We prove the natural extension that *a compact semiring without zeroid is a ring.* As a by-product, we obtain a generalization for the commutative case of Numakura's theorem that *a compact semigroup satisfying the cancellation law is a group* [4] to *a compact abelian semigroup without zeroid is a group.*

I. Kaplansky [2] has given structure theorems for compact rings. He proved that *a compact semi-simple ring is isomorphic and homeomorphic to a Cartesian direct sum of finite simple rings* [2]. Hence, this structure theorem remains true for a compact semi-simple semiring.

Only semirings with commutative addition and a zero, in the sense of Vandiver and Weaver [5], are considered. This paper has benefited materially from discussion with H. Zassenhaus of the University of Notre Dame.

2 *Quotient spaces. Definition* 1. A topological semiring is a semiring S together with a Hausdorff topology on S under which the semiring operations are continuous. Since the zeroid of a semiring will play an important role in what follows, we repeat its definition.

Definition 2. The zeroid $Z(S)$ of a semiring S is the set of elements z of S for which the equation $z+x=x$ is solvable in S.

In a previous paper [1] we defined two elements i_1, i_2 of a semiring S to be equivalent if the equation $i_1+x=i_2+x$ is solvable in S. These equivalence classes i^* represented by $i\in S$ form a semiring S^* with cancellation law of addition, according to the laws $i_1^*+i_2^*=(i_1+i_2)^*$, $i_1^*i_2^*=(i_1i_2)^*$. S^* is then a halfring [6]. The zeroid consists of all elements z of S for which $z^*=0$, i.e. the zeroid of S is the inverse image of the O-element of S^* under the homeomorphism $i\rightarrow i^*$ of S onto S^*.

We introduce in S^* the quotient topology, that is the largest topology for S^* such that the function $i\rightarrow i^*$ is a continuous mapping of S onto S^*. We assume that S is a compact space. Then S^* is also compact space, for the function $i\rightarrow i^*$ is a continuous mapping of S onto S^* [3].

LEMMA 1. *The compact space S^* is Hausdorff.*

Proof. We recall the following theorems: *Let X be a topological*