124. On Singular Perturbation of Linear Partial Differential Equations with Constant Coefficients. II

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§0. Introduction. Professor M. Nagumo proved in his recent note[1] the following theorem on the stability of linear partial differential equations of the form

(0) $$L_\varepsilon(u)=\sum_{\mu=0}^{l} P_\mu(\partial_x, \varepsilon)\partial_t^\mu u=f_\varepsilon(t, x).\text{[2]}$$

Definition. We say that the equation (0) is *H_p-stable for $\varepsilon\downarrow 0$ in* $0\leqq t\leqq T$ with respect to a particular solution $u=u_0(t)$ of (0) for $\varepsilon=0$, if $u_\varepsilon(t)\to u_0(t)$ in $H_{p,x}$ uniformly for $0\leqq t\leqq T$, whenever $f_\varepsilon(t, x)\to f_0(t, x)$ in $H_{p,x}$ uniformly for $0\leqq t\leqq T$, and $u_\varepsilon(t)=u(t, x, \varepsilon)$ is a generalized H_p-solution of (0) such that $\partial_t^{j-1}u_\varepsilon(0)\to\partial_t^{j-1}u_0(0)$ in $H_{p,x}$ $(j=1,\cdots, l)$.

Theorem A. *Let degree of $\{P_\mu(\xi, \varepsilon)-P_\mu(\xi, 0)\}\leqq k$ $(\mu=0,\cdots, l)$ and let $u=u_0(t)$ be an l-times continuously $H_{p+k,x}$-differentiable solution of (0) for $\varepsilon=0$ in $0\leqq t\leqq T$. In order that (0) be H_p-stable for $\varepsilon\downarrow 0$ with respect to $u=u_0(t)$ in $0\leqq t\leqq T$, it is necessary and sufficient that there exist constants $\varepsilon_0>0$ and C such that:*

$$\sup_{\xi\in E^m} Y_j(t, \xi, \varepsilon)\leqq C \quad \textit{for}\ \ 0\leqq t\leqq T,\ 0<\varepsilon\leqq\varepsilon_0$$

and

$$\sup_{\xi\in E^m}\int_0^T |P_l(\xi, \varepsilon)^{-1}Y_l(t, \xi, \varepsilon)|\,dt\leqq C \quad \textit{for}\ \ 0<\varepsilon\leqq\varepsilon_0$$

where $Y=Y_j(t, \xi, \varepsilon)$ are matricial solutions of

$$\sum_{\mu=0}^{l} P_\mu(i\xi, \varepsilon)(d/dt)^\mu y=0$$

with the initial conditions $\partial_t^{k-1}Y_j(0, \xi, \varepsilon)=\delta_{jk}1$ $(k=1,\cdots, l)$.

In this note we are concerned with the H_p-stability of the equation

$$\varepsilon\cdot\partial_t^2 u+a\cdot\partial_t u+Q(\partial_x)u=f_\varepsilon(t, x)$$

where a is a complex constant and $Q(i\xi)$ is a polynomial in $\xi\in E^m$, and making use of Theorem A we decide the structure of $Q(i\xi)$ in order that this equation be H_p-stable.[3]

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§1. Main theorems. In this section we shall exhibit three theorems on H_p-stability of the equation

(1.1) $$\varepsilon\cdot\partial_t^2 u+a\cdot\partial_t u+Q(\partial_x)u=f_\varepsilon(t, x).$$

The fundamental solutions of the equation

$$\varepsilon(d^2/dt^2)y+a(d/dt)y+Q(i\xi)y=0$$

are represented by

1) M. Nagumo: On singular perturbation of linear partial differential equations with constant coefficients. I, Proc. Japan Acad., **35**, 449 (1959).

2) We use the same notations and terminology with Nagumo 1).

3) In this note we say H_p-stable for simplicity.