139. On Probabilities of Non-Paternity with Reference to Consanguinity. I

By Yûsaku KOMATU and Han NISHIMIYA

Department of Mathematics, Tokyo Institute of Technology and
Department of Mathematics, Shibaura Institute of Technology

(Comm. by T. FURUHATA, M.J.A., Dec. 12, 1959)

In a series of papers we have dealt with an inherited character subject to Mendelian law from probabilistic view point. Considering a single inherited character which consists of m multiple alleles at one diploid locus denoted by

$$A_i \qquad (i=1,\cdots,m),$$

we have determined, among others, the probability of non-paternity, i.e. the probability that a putative man has a genotype inconsistent with those of a mother-child combination, both the man and the combination being chosen at random from a population; cf. esp. [1]. It has been supposed that there exists no consanguineous relationship among the putative man and the parents of the child. Accordingly, the distribution of genotypes in the population has been supposed to be an ordinary one in an equilibrium state.

On the other hand, in another series of papers we have discussed the distributions of genotypes in several definite combinations which consist of individuals chosen at random under various imposed consanguineous relationships. In particular, we have shown that the probabilities of sib-combinations are to be modified according to the presence of consanguinity; cf. esp. [2].

Now the question arises to determine how the probabilities of non-paternity are to be modified correspondingly. In the present paper this problem will be studied for a few particular types of consanguinity existent among a triple of relevant individuals. Namely, we restrict ourselves here to consider triples for which (μ, ν)th sibship exists between parents of a child or between a putative man and one of the parents. A remarkable monotoneity caused by the presence of consanguinity will become clear while it seems previously plausible to some extent. In fact, it will be verified that in every case the probability of non-paternity always decreases by virtue of the presence of consanguinity. According to circumstances, each kind of consanguineous relationships will be considered separately in the sequel.

Various definitions and notations concerning several concepts contained in the previous papers will be retained here also. In particular, the frequencies of the genes A_i $(i=1,\cdots,m)$ will be denoted often