94. On Osima's Blocks of Group Characters

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Let $\mathfrak{G}$ be a group of finite order g and p be a fixed rational prime. M. Osima, in his earlier paper [4], introduced a concept of blocks of characters with regard to a subgroup $\mathfrak{H}$ of $\mathfrak{G}$ ("$\mathfrak{H}$-blocks"). Let $\mathfrak{H}_0$ be the maximal normal subgroup of $\mathfrak{G}$ contained in $\mathfrak{H}$. It is well known that the irreducible characters[1] $\phi_1, \phi_2, \cdots, \phi_k$ of $\mathfrak{H}_0$ are distributed into the classes $\mathfrak{B}_1, \mathfrak{B}_2, \cdots, \mathfrak{B}_s$ of associated characters in $\mathfrak{G}$. If $\mathfrak{B}'_1, \mathfrak{B}'_2, \cdots, \mathfrak{B}'_{s'}$ are the classes of associated irreducible characters of $\mathfrak{H}_0$ in $\mathfrak{H}$, then each class $\mathfrak{B}_\sigma$ is a collection of classes $\mathfrak{B}'_\rho$. Let $\chi_1, \chi_2, \cdots, \chi_n$ be the irreducible characters of $\mathfrak{G}$ and $\theta_1, \theta_2, \cdots, \theta_h$ be those of $\mathfrak{H}$. As is well known, there corresponds to each character χ_i exactly one class $\mathfrak{B}_\sigma$ such that

$$\chi_i(H_0) = s_{i\sigma} \sum_{\phi_\mu \in \mathfrak{B}_\sigma} \phi_\mu(H_0) \qquad (H_0 \in \mathfrak{H}_0)$$

where $s_{i\sigma}$ is a positive rational integer. If a class $\mathfrak{B}_\sigma$ corresponds to a character χ_i in this sense, we say that χ_i belongs to $\mathfrak{B}_\sigma$ by counting χ_i in $\mathfrak{B}_\sigma$. We also say that θ_λ belongs to $\mathfrak{B}_\sigma$ if θ_λ belongs to $\mathfrak{B}'_\rho$ contained in $\mathfrak{B}_\sigma$. Then the classes $\mathfrak{B}_\sigma$ are the $\mathfrak{H}$-blocks of $\mathfrak{G}$ in Osima's sense. From the definition, we see that χ_i and χ_j belong to the same $\mathfrak{H}$-block of $\mathfrak{G}$ if and only if $\chi_i(H_0)/\chi_i(1) = \chi_j(H_0)/\chi_j(1)$ for all elements H_0 of $\mathfrak{H}_0$ [4], where 1 denotes the identity of $\mathfrak{G}$.

In the following, "block" of a group will always mean block with regard to a p-Sylow subgroup of the group. While Brauer's blocks for a rational prime q will be referred always as q-blocks. The purpose of this paper is to consider a connection between blocks of $\mathfrak{G}$ and the blocks of the normalizer $\mathfrak{N}(R)$ of a p-regular element R in $\mathfrak{G}$.

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1. Let $\mathfrak{P}$ be a p-Sylow subgroup of $\mathfrak{G}$ and $\mathfrak{P}_0$ be the maximal normal p-subgroup of $\mathfrak{G}$. We shall denote by $\mathfrak{B}_1, \mathfrak{B}_2, \cdots, \mathfrak{B}_s$ the blocks of $\mathfrak{G}$ with regard to $\mathfrak{P}$. For each $\mathfrak{B}_\sigma$ we set

$$\Delta_\sigma = \sum_{\chi_i \in \mathfrak{B}_\sigma} e_i, \tag{1.1}$$

where e_i is the primitive idempotent of the center Z of the group ring of $\mathfrak{G}$ over the field Ω of g-th roots of unity which belongs to χ_i. Let $K_1, K_2, \cdots, K_n$ be the classes of conjugate elements in $\mathfrak{G}$ and G_1,

1) The term "irreducible character" will always mean absolutely irreducible ordinary character.