36. On the Normal Basis Theorem of the Galois Theory for Finite Factors

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(Comm. by K. KUNUGI, M.J.A., March 13, 1961)

This paper is a continuation of [3], [4] and [5]. In these preceding papers we have shown that the fundamental principles of the Galois theory remain true for finite factors as same as for rings with minimum condition. In this paper we shall show the existence of a normal basis for a Galois extension of a finite factor and a theorem concerning to normal subgroups of a Galois group corresponding to the well-known theorems of the classical theory.

1. Employing the terminology of J. Dixmier [1], we denote by A a continuous finite factor standardly acting on a separable Hilbert space H and by G a finite group of outer automorphisms of A. Then G permits a unitary representation $\{u_g\}$ on H such that $x^g=u_g^*xu_g$ for $x\in A$.[1] Putting $x'^g=u_g^*x'u_g$ for $x'\in A'$, every $g\in G$ $(g\neq 1)$ induces an outer automorphism to the commutor A' of A. Hence G may be considered as a group of outer automorphisms of A' as well as of A (cf. [4]). We put B and $\hat{B}$ the subfactors of A and A' consisting of all invariant elements by G respectively.

Next we construct the crossed product $G\otimes A$ of the factor A by the group G (cf. [2]). By the outer property of G, $G\otimes A$ is a finite factor. To say more precisely, let us denote by $G\otimes H$ the Hilbert space of all functions defined on G taking values in H. Let $\sum_g g\otimes\varphi_g$ be a function belonging to $G\otimes H$ which takes value $\varphi_g\in H$ at $g\in G$, then every $a\in A$ and $g_0\in G$ defines an operator $a^\#$ and $g_0^\#$ on $G\otimes H$ such that

$$(\textstyle\sum_g g\otimes\varphi_g)a^\#=\sum_g g\otimes\varphi_g a,\quad (\sum_g g\otimes\varphi_g)g_0^\#=\sum_g gg_0\otimes\varphi u_{g_0}$$

respectively. The crossed product $G\otimes A$ can be understand as a von Neumann algebra generated by $\{a^\#, g_0^\# \mid a\in A, g_0\in G\}$. We put $A^\#=\{a^\# \mid a\in A\}$, then by the construction of the Hilbert space $G\otimes H$ and the definition of $a^\#$, we can easily understand that $A^\#$ is an n-fold copy of A acting on H. Since the order n of G is finite, both $A^\#$ and its commutor $A^{\#\prime}$ are finite factors.

As discussed in [4], there exists a subspace K of $G\otimes H$ invariant by $G\otimes A$, on which the restriction of $G\otimes A$ is unitarily isomorphic to $\hat{B}'$ acting on H. In fact, let $1^e\in H$ be a trace element of A sat-

1) x^g is the image of x due to an automorphism g.