114. Note on the Direct Product of Certain Groupoids[1]

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Consider a semigroup G satisfying

(1.1) There is at least one (left identity) $e \in G$ such that $ea=a$ for all $a \in G$.

(1.2) For any $a \in G$ and for any left identity $e \in G$ there is at least one $b \in G$ such that $ab=e$.

A. H. Clifford [1] and H. B. Mann [2] investigated such systems and they obtained the same result: the system is the direct product of a right singular semigroup and a group. Clifford called such systems multiple groups, Mann called them (l, r) systems, but we call them right groups. In this note we shall define an M-groupoid as generalization of right groups and shall study the conditions for M-groupoids.

DEFINITION. An M-groupoid S is a groupoid[2] (Bruck [4]) which satisfies the following conditions:

(2.1) There is at least one $e \in S$ such that $ex=x$ for all $x \in S$.

(2.2) If y or z is a left identity of S, then $(xy)z=x(yz)$ for all $x \in S$.

(2.3) For any $x \in S$ there is a unique left identity e (which may depend on x) such that $xe=x$.

THEOREM 1. *An M-groupoid S is the direct product of a right singular semigroup and a groupoid with a two-sided identity, and conversely.*

For the proof of this theorem we use the following lemma:

LEMMA. *If and only if a groupoid S has two orthogonal decompositions, it is isomorphic to the direct product of the two factor groupoids obtained from the two decompositions.*

Clifford introduced the notation "orthogonal decomposition" in his paper [1], p. 869, but he did not apply the principle directly. Although this lemma is obvious according to K. Shoda [3], p. 158, we can easily prove it with elementary method.

DEFINITION. A right group S is a groupoid which satisfies the following conditions:

(3.1) For any $x, y, z \in S$, $(xy)z=x(yz)$

(3.2) For any $a, b \in S$, there is a unique $c \in S$ such that $ac=b$.

1) The detail proof will be given elsewhere.

2) A groupoid is a system in which a binary operation is defined.