101. On the Existence of Periodic Solutions of Difference-Differential Equations

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In a difference-differential equation

$$x'(t+1)=ax(t+1)+bx(t)+w(t), \tag{1}$$

we suppose that a and b are constant, and $w(t)$ is a continuous and periodic function of the period ω for $-\infty<t<\infty$.

Let $K(t)$ be a kernel function of (1), that is, a solution of (1) under the conditions $K(t)=0$ $(-1\leqq t<0)$, $K(0)=1$, and $w(t)\equiv 0$.

In the sequel, the following condition is always supposed: *every real part of all the roots of the characteristic equation*

$$e^s(s-a)-b=0$$

is less than $-\delta$, *where* δ *is a positive constant.*

Then, $K(t)$ satisfies the equations

$$K'(t+1)=aK(t+1)+bK(t) \qquad (0<t<\infty),$$
$$K'(t)=aK(t) \qquad (0<t<1)$$

and the inequality

$$|K(t)|\leqq ce^{-\delta t} \qquad (0\leqq t<\infty).$$

If we define a function $p(t)$ such that

$$p(t+1)=\int_{-\infty}^{t} w(s)K(t-s)ds, \tag{2}$$

we find that $p(t)$ is a periodic solution of (1) of the period ω, if we formally differentiate (2) and use the periodicity of $w(t)$. This is the fundamental idea in the following discussions.

The purpose of this paper is to discuss the existence of periodic solutions of the equation (1) which has a term $f(t,x,y,\mu)$ or $\mu f(t,x,y)$ instead of $w(t)$. We will establish the following theorems.

THEOREM 1. *In the equation*

$$x'(t+1)=ax(t+1)+bx(t)+f(t,x(t+1),x(t)), \tag{3}$$

where a *and* b *are constant, we suppose that* $f(t,x,y)$ *satisfies the following conditions;*

(i) $f(t,x,y)$ *is continuous for any* t,x,y *and* $f(t,0,0)$ *does not identically vanish;*

(ii) $f(t,x,y)$ *is a periodic function of* t *of the period* ω, *where* ω *is a positive constant;*

(iii) $f(t,x,y)$ *satisfies Lipschitz condition such that*