142. *Evolutional Equations of Parabolic Type*

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1. **Introduction.** The object of this note is to state some theorems concerning the existence and the uniqueness of the solution of the initial value problem for the evolutional equation

$$dx(t)/dt = A(t)x(t) + f(t), \quad a \leqq t \leqq b. \tag{1.1}$$

Here the unknown $x(t)$ as well as the inhomogeneous term $f(t)$ is a function on the closed interval $[a, b]$ to a Banach space X, whereas $A(t)$ is a function on $[a, b]$ to the set of (in general unbounded) linear operators acting in X.

For each $t, A(t)$ is assumed to be the infinitesimal generator of an analytic semi-group of bounded operators. We make some additional assumptions on the resolvents of $A(t)$. However, we do not assume that the domain of some fractional power of $A(t)$ is independent of t.

Under these assumptions, we will construct the *evolution operator* (or *fundamental solution*) $U(t, s)$, defined for $a \leqq s \leqq t \leqq b$, such that the solution of (1.1) can be expressed in the form

$$x(t) = U(t, s)x(s) + \int_s^t U(t, \sigma)f(\sigma)d\sigma. \tag{1.2}$$

2. **Notations and assumptions.** We denote by $\sum$ the closed angular domain consisting of all the complex numbers λ satisfying $|\arg \lambda| \leqq \pi/2 + \theta$, where θ is a fixed angle with $0 < \theta < \pi/2$. We make the following assumptions.

(A.1) For each $t \in [a, b], A(t)$ is a densely defined, closed linear operator whose resolvent set $\rho(A(t))$ contains $\sum$.

(A.2) There exists a positive constant M such that the resolvent of $A(t)$ satisfies

$$\|(\lambda I - A(t))^{-1}\| \leqq M/|\lambda|, \tag{2.1}$$

for each $t \in [a, b]$ and $\lambda \in \sum$.

(A.3) $A(t)^{-1}$, which is a bounded operator valued function of t, is once Hölder continuously differentiable in $a \leqq t \leqq b$:

$$\|dA(t)^{-1}/dt - dA(s)^{-1}/ds\| \leqq K|t-s|^{\alpha}, \; K, \alpha > 0. \tag{2.2}$$

(A.4) There exist positive constants N and ρ with $0 \leqq \rho < 1$, such that

$$\left\| \frac{\partial}{\partial t}(\lambda I - A(t))^{-1} \right\| \leqq \frac{N}{|\lambda|^{1-\rho}}, \tag{2.3}$$

for each $t \in [a, b]$ and $\lambda \in \sum$.

In what follows, we denote by C constants which depend only on the constants appearing in the above assumptions.