21. *Multipliers of Banach Algebras*

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1. For a commutative semi-simple Banach algebra A, regarding as an algebra of continuous functions on the space X of all maximal regular ideals of A, a multiplier g of A is introduced by Helgason [2] as a function on X satisfying

(1) $$gA \subseteq A.$$

The notion of multiplier is recently generalized by Wang [3] when A is a commutative Banach algebra "without order" in the sense that $aA=0$ implies $a=0$: A map g of A into A is called a multiplier if g satisfies

(2) $$(ga)ba=(gb),$$

or equivalently,

(3) $$g(ab)=(ga)b,$$

for any a and b in A. A similar observation is also given by Foias [1] who used "factor function" instead of multiplier and rather restrictive conditions on A. The definition is equivalent to that of Helgason if A is semi-simple. Foias and Wang proved, among others, the following

THEOREM 1. *The multiplier algebra $M(A)$, the set of all multipliers of A, is a Banach algebra having the identity and closed with respect to the strong operator topology.*

In non-commutative case, (2) is not equivalent to (3). However, it is reasonable to expect that a linear operator g defined by (3) plays some role even in non-commutative case, since an endomorphism g satisfying (3) is known as an admissible A-endomorphism in the theory of classical rings. In the below, it will be shown that Theorem 1 is also true for a non-commutative Banach algebra.

2. A (*left*) *multiplier* of a (not necessarily commutative) Banach algebra A is a (bounded) linear operator g which maps A into A satisfying (3). By this definition, it is obvious that $M(A)$ forms a normed algebra with the identity by the operator norm, since $g,f \in M(A)$ implies

$$fg(ab)=f[(ga)b]=(fga)b.$$

If g_α converges strongly to a linear operator g, then

$$g(ab)=\lim_\alpha g_\alpha(ab)=\lim_\alpha(g_\alpha a)b=(ga)b$$

shows that g belongs to $M(A)$, whence $M(A)$ is strongly closed. Naturally, $M(A)$ is complete with respect to the operator norm.