26. *Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces.* VI

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On the assumption that ζ and Ω denote respectively a given complex number and an appropriately large circle with center at the origin and that the ordinary part $R(\lambda)$ of the function $S(\lambda)$ defined in the statement of Theorem 1 [1] is a transcendental integral function, in this paper we shall discuss the relation between the distribution of ζ-points of $S(\lambda)$ and that of ζ-points of $R(\lambda)$ in the exterior of the same circle Ω and shall then show that, if each of $S(\lambda)$ and $R(\lambda)$ has its finite exceptional value for the exterior of Ω, the two exceptional values are identical under some conditions.

Theorem 16. Let $S(\lambda)$, $R(\lambda)$, and $\{\lambda_\nu\}$ be the same notations as those in Theorem 1; let σ be an appropriately large number such that $\sup_\nu |\lambda_\nu| < \sigma < \infty$; let $\{z_n\}$ be an infinite sequence of all ζ-points of $R(\lambda)$ in the exterior of the circle $|\lambda| = \sigma$ such that

$$\left.\begin{array}{l} R(z_n)=\zeta \\ \sigma<|z_n|\leqq|z_{n+1}| \end{array}\right\} \quad (n=1, 2, 3, \cdots)$$

and $|z_n| \to \infty$ $(n \to \infty)$, each ζ-point being counted with the proper multiplicity; let

$$C = \sup_n \left\{ \frac{1}{2\pi} \left| \int_0^{2\pi} S(\rho e^{it}) e^{int} dt \right| \right\} \quad (<\infty),$$

where ρ is an arbitrarily prescribed number subject to the condition $\sup_\nu |\lambda_\nu| < \rho < \infty$; let μ be the greatest value of the positive integers ν_n in the first non-zero coefficients $R^{(\nu_n)}(z_n)/\nu_n!$ of the Taylor expansions of $R(\lambda)$ at z_n, $n=1, 2, 3, \cdots$; let $m \equiv \inf_n \{|R^{(\nu_n)}(z_n)|/\nu_n!\}$ be positive; let $M \equiv \sup_n [\max_k \{|R^{(k)}(z_n)|/k!\}]$ $(n, k=1, 2, 3, \cdots)$ be finite; and let r be an arbitrarily given number such that $0 < r < m/(M+m)$. Then, in the interior of the circle $|\lambda - z_n| = r$ associated with any z_n satisfying

$$\left\{ \frac{C}{r^\mu \left(m - \dfrac{Mr}{1-r} \right)} + 1 \right\} \rho + r < |z_n|,$$

S(λ) has ζ-points whose number (counted according to multiplicity) equals that of ζ-points of $R(\lambda)$ in the interior of the same circle as it.

Proof. It must first be noted that the case where $R(\lambda)$ has such ζ-points $\{z_n\}$ as was described in the statment of the present theorem