146. *Eigenfunction Expansion Associated with the Operator $-\Delta$ in the Exterior Domain*

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1. Introduction. An attempt to use distorted plane waves for expanding an arbitrary function which is square integrable was first carried through by Ikebe in 1960 [1]. He treated the Schroedinger operator $-\Delta+q(x)$ in the whole 3-dimensional Euclidean space E, where Δ denotes Laplacian and $q(x)$ is a potential function. In the present paper we consider the similar problem for the Schroedinger operator of another type, i.e., of a rigid body. This means that no potential exists, but negative Laplacian has a boundary condition on some bounded, smooth and closed surface representing the rigid body. Naturally the space with which we are concerned is not the whole 3-dimensional Euclidean space but the exterior domain of the surface. The method used is essentially the same to the Ikebe's one, except for the use of the potential theory which seems indispensable in our case. No explicit mention is made of the smoothness of the surface, for it is rather complicated. The reader will find it in any textbook on the potential theory.[1)]

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2. Exterior Dirichlet problem. S denotes a sufficiently smooth, closed and bounded surface in E. Ω is the exterior domain relative to S. Suppose that $u(x)$ satisfies

(2.1) $$(-\Delta-\kappa^2)u(x)=0, \qquad x\in\Omega,$$

(2.2) $$u(p)=f(p), \qquad p\in S,$$

(2.3) $$\frac{\partial u}{\partial r}-i\kappa u=e^{-\beta r}o(r^{-1}),$$
$$u(x)=e^{-\beta r}O(r^{-1}),\ \beta=Im\ \kappa.^{2)}$$

Then the function is called the solution of the exterior Dirichlet problem for the boundary condition $f(p)$. Then we can show the

1) See e.g. [8]. We must also suppose, as is usually supposed in the potential theory, for the surface that the following inequality holds.

$$\int\frac{|\cos(n_p,\overrightarrow{xp})|}{|x-p|^2}dS_p\le C.$$

Here C denotes a positive constant independent of x.

2) Sommerfeld's radiation condition and finiteness condition in the generalized form. See [3].