141. On the Cauchy Problem for a Class of Multicomponent Diffusion Systems

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Introduction. This note discusses the Cauchy problem for a class of multicomponent diffusion systems of the form

$$\begin{aligned} \Lambda u &= f(x, t, u, v), \\ \partial v/\partial t &= g(x, t, u, v), \end{aligned} \tag{1}$$

where $x=(x_1,\cdots,x_n)$ and Λ is a linear parabolic differential operator:

$$\Lambda u \equiv \partial u/\partial t - \left[\sum_{i,j=1}^{n} a_{ij}(x,t)\partial^2 u/\partial x_i \partial x_j + \sum_{i=1}^{n} b_i(x,t)\partial u/\partial x_i + c(x,t)u\right].$$

Let E^n denote the n-dimensional Euclidean x-space and H the strip $H=E^n\times(0, T]$, $T>0$, in the $(n+1)$-dimensional (x, t)-space.

By the Cauchy problem in question we mean the problem of finding function pairs $\{u(x, t),\ v(x, t)\}$ which are continuous in $\overline{H}$, satisfy the system (1) in H and take on the given initial values:

$$u(x, 0)=\varphi(x) \quad v(x, 0)=\psi(x), \quad x\in E^n. \tag{2}$$

Our main concern in this note is with the comparison (§1) and the existence (§2) of solutions of the problem (1)–(2), being suggested by an elegant work of A. McNabb [1] on the first boundary value problem for the system (1) in cylindrical domains.[1)]

Preliminary hypotheses. The following assumptions concerning the system (1) will be made throughout the note:

1) *The coefficients a_{ij}, b_i and c are defined and continuous in $\overline{H}$;*

2) *At each point $(x, t)\in\overline{H}$ and for all real n-tuples $\xi=(\xi_1\cdots,\xi_n)$,*

$$\sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \geqq a_0\sum_{i=1}^{n}\xi_i^2, \quad (a_0\text{: a positive constant}); \tag{3}$$

3) *The functions f and g are defined in the domain $\mathcal{D}=\{(x,t)\in\overline{H},\ -\infty<u<\infty,\ -\infty<v<\infty\}$ and are subject to the conditions:*

i) *f is a non-decreasing function of v, while g is a non-decreasing function of u;*

ii) *Both f and g are uniformly Lipschitz continuous relative to u and v:*

$$|h(x,t,u,v)-h(x,t,\overline{u},\overline{v})| \leqq M(|u-\overline{u}|+|v-\overline{v}|), \tag{4}$$

for (x,t,u,v), $(x,t,\overline{u},\overline{v})\in\mathcal{D}$ with $h=f$ or g.

§1. Comparison theorems. To begin with, the following spaces of function pairs $\{u(x, t\},\ v(x, t)\}$ defined in $\overline{H}$ are introduced.

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1) We also refer to the works of V. N. Maslennikova [2], [3].