165. On the Uniqueness of Balayaged Measures

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Introduction. Let Ω be a locally compact Hausdorff space, every compact subset of which is separable, and $G(x, y)$ be a positive continuous (in the extended sense) kernel on Ω.[1] In [2], we proved that a regular symmetric balayable kernel G satisfies the U- and BU-principles[2] if and only if it is non-degenerate, that is, for any different points x_1 and x_2 in Ω,

$$G(x, x_1)/G(x, x_2) \fallingdotseq \text{any constant in } \Omega.$$

In this paper we shall extend this result to non-symmetric kernels.

§1. Non-degeneracy. Theorem 1. *If G satisfies the U- or BU-principle, it is non-degenerate.*

This is evident.

Theorem 2. *Let G be non-degenerate and satisfy*

(i) *the domination principle or*

(ii) *the balayage principle and the continuity principle. Then its adjoint kernel $\check{G}$ is non-degenerate.*

Proof. If G satisfies the condition (ii), then it satisfies (i).[3] Therefore we may assume that G satisfies the domination principle. Contrary suppose that $\check{G}$ is degenerate. Then there are different points x_1 and x_2 such that $G(x_1, x) = aG(x_2, x)$ for any point x in Ω with a positive constant a. Then $G(x_1, x_1)$ and $G(x_2, x_2)$ are finite and $G\varepsilon_{x_1}(x_i) = bG\varepsilon_{x_2}(x_i)$ $(i=1, 2)$ with $b = G(x_1, x_1)/G(x_1, x_2)$. Hence by the domination principle

$$G\varepsilon_{x_1} = bG\varepsilon_{x_2} \quad \text{in } \Omega.$$

This shows that G is degenerate.

Corollary. *The adjoint kernel $\check{G}$ is non-degenerate if and only if G is non-degenerate, provided that*

(i) *G satisfies the domination principle and $\check{G}$ satisfies the continuity principle or*

(ii) *G satisfies the balayage principle and the continuity principle.*

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1) We use the same notations as in [3].

2) The U-principle means that if G-potentials of positive measures with compact support coincide with each other G-p.p.p. in Ω, then the measures are identical.

The BU-principle means that the G-balayaged measure is uniquely determined by a given positive measure and a compact set.

3) Cf. [3, 4].