72. *Product of Minimal Topological Spaces*

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A topological space $(X, \mathfrak{T})$ is said to be minimal Hausdorff if $\mathfrak{T}$ is Hausdorff and there exists no Hausdorff topology on X strictly weaker than $\mathfrak{T}$. In the same way, a topological space $(X, \mathfrak{T})$ is said to be minimal regular if $\mathfrak{T}$ is regular and there exists no regular topology on X strictly weaker than $\mathfrak{T}$. These definitions are due to M. P. Berri and R. H. Sorgenfrey ([1], [2]).

It is the purpose of this note to give an affirmative answer to the question, "Is the topological product of minimal Hausdorff spaces necessarily minimal Hausdorff ?", which is one of the problems which are not yet solved in the paper [1]. The converse of this question is already proved in [1], namely, if the nonempty product space is minimal Hausdorff, then each factor space is minimal Hausdorff.

We shall next obtain some results concerning minimal regular spaces.

§1. **Product of minimal Hausdorff spaces.** The following facts which are concerned with minimal Hausdorff spaces have been shown in [1].

A necessary and sufficient condition that a Hausdorff space $(X, \mathfrak{T})$ be minimal Hausdorff is that $\mathfrak{T}$ satisfies property S:

$S(1)$ Every open filter-base (which composed exclusively of open sets) has an adherent point:

$S(2)$ If an open filter-base has a unique adherent point, then it converges to this point.

A Hausdorff space X which satisfies $S(2)$ also satisfies $S(1)$.

From [3, p. 110], property $S(1)$ is a necessary and sufficient condition for a Hausdorff space to be absolutely closed.

Theorem 1. *The topological product of minimal Hausdorff spaces is minimal Hausdorff.*

Proof. Let $Z=\prod_{\alpha} X_\alpha$, where X_α is a minimal Hausdorff space for all α. Let $\mathfrak{F}$ be an open filter-base with a unique adherent point $z^0=(x^0_\alpha)\in Z$. We shall show that $\mathfrak{F}$ converges to the point z^0. In order to show this, we divide the proof into three parts.

1) For every α, let $\mathfrak{F}_\alpha=\{F_\alpha=\mathrm{Pro}_\alpha F \mid F\in\mathfrak{F}\}$,*) then $\mathfrak{F}_\alpha$ is an open filter-base in X_α.

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*) $\mathrm{Pro}_\alpha F$ denotes the projection of F into the factor space X_α.