71. On Some Singular Integral Equations. I

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1. The theory of linear singular line integral equations with Cauchy-type kernel, on which extensive work has been done, is already a classical one. Beautiful unified results have been published in Muskhelishvili's book [1]; however, work is still being done on this theory and on that of some nonlinear line equations. Muskhelishvili worked on a linear line singular integral equation of the second kind first, reducing it to the Hilbert problem, and solved the "dominant" equation in which the kernel is $1/(t-t_0)$. With regard to a general equation of the second kind, he says only that it is reduced to a Fredholm integral equation by the application of the solution for the dominant equation, i.e., of the inverse operator of the dominant operator, on the general equation of the second kind. Therefore, no compact formulation is obtained by this method for a solution of a general equation. The theory of equations of the first kind, in his method, is included in that of equations of the second kind as a special case, and no compact formulation for a solution is given.

The author encountered an equation of the first kind while working on certain Dirichlet and Neumann problems for the wave equation [2], but he solved it by a revised form of Muskhelishvili's method. The reasons why he was not satisfied with Muskhelishvili's method are as follows: (1) Consideration of an equation of the second kind first results in unnecessary complication of the method, and (2) for the purpose of solving a singular integral equation, it is not necessary to investigate the Hilbert problem as precisely as Muskhelishivili did if an equation of the first kind is solved first. In this paper it will be shown first how to derive a compact formulation for solutions of a linear, singular line equation of the first kind directly, without referring to a Fredholm integral equation, and then how to derive a compact formulation for the solution of an equation of the second kind by reducing it to an equation of the first kind. A general case will be treated, in which the path of integration L is a mixture of closed contours and arcs.

The method will be generalized to the cases of a singular surface integral equation and of some nonlinear integral equations in the

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