68. *Electromagnetic Field in a Domain Bounded by Coaxial Circular Cylinders with Slots*

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(Comm. by Kinjirô KUNUGI, M.J.A., May 9, 1964)

1. In this paper, the exact solution of the Maxwell equations is derived by means of dual series equations and a singular integral equation with Cauchy kernel for a domain exterior to the inner member of a pair of coaxial circular cylinders where the outer member has a finite number of axial slots of infinite length and arbitrary width and in the presence of arbitrary axial line sources. This is a canonical form of some problems in radio engineering which became of interest recently. There are many works done on slotted cylinders [1], [2]. However, in these works, distribution of the field at the slot is usually assumed to be given or is approximated by a known distribution, say, by that of a static field at a slit on a plane. Recently, some work has been done [3] on waveguides by the method of singular integral equations.

In this paper, a compact formulation for the field components will be given which satisfies all required conditions, i.e., the boundary condition, the radiation condition, the edge condition at the edges of the slots and the continuity condition of the field at the slots.

2. Suppose that the expressions

$$r=a,\quad 0\leqq\phi\leqq 2\pi,\quad -\infty<z<\infty,$$
$$r=b,\quad \alpha_j<\phi<\beta_{j+1},\quad -\infty<z<\infty,$$
$$(0<a<b,\ \beta_j<\alpha_j<\beta_{j+1},\ \beta_{\nu+1}=\beta_1,\ j=1,2,\cdots,\nu)$$

represent a pair of coaxial circular cylinders of perfect conductivity, with ν slot specified by

$$r=b,\quad \beta_j<\phi<\alpha_j,\quad -\infty<z<\infty.$$

Without loss of generality, we can assume that there is one axial line source in the interior ($a\leqq r<b$) at Q_i: $r=r_i, \phi=\phi_i$, and one axial line source in the exterior ($b<r$) at $Q_e: r=r_e$, $\phi=\phi_e$, because fields for more sources are obtainable by the principle of superposition. In this case, it is easy to see that the Maxwell equations $\nabla\times E=-i\omega\mu H$, $\nabla\times H=i\omega\varepsilon E$ are equivalent to

$$\Delta u+k^2u=0 \tag{1}$$

with the boundary conditions

$$u=0\quad\text{at}\quad r=a\quad\text{and}\quad r=b,\ \alpha_j<\phi<\beta_{j+1}, \tag{2}$$

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