84. C^* Algebra and its Extension as the Set of Observables

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§1. **Introduction.** The operator A having the bounded usual expectation value $\langle \Phi, A\Psi \rangle$ for any two states Φ and Ψ is a bounded operator.

These bounded operators construct C^* algebra which is considered as the set of observables. It seems to us that the name of observables is derived from the reason such that it has always bounded usual expectation value. (Namely, it can be observed.)

On the other hand the most of the quantities appearing in quantum field theory are unbounded operators such as field function, creation operator and annihilation operator etc. The set of unbounded operators is also investigated by John Von Neumann [2], [3]. But the topology to give the relation between the bounded operators and the unbounded operators is not to be seen in his work. Using spectral decomposed form we can obtain the series of bounded operators approached to the self adjoint unbounded operator. But it is difficult to treat concretely the unbounded operators using above method. To treat unbounded operators concretely, weak closure of the set of bounded operators is used by R. Kastler and K. Haag in [1]. But his topology is too strong to extend the set of observables. Using the weak topology related to a fixed dense subspace, this difficulty is eliminated temporarily and unnaturally [3].

In this paper, we show the defect of extension in [1], and extend truely the set of observables by using E. R. Integral [4], considering the various methods.

§2. **Observable.** Abstruct C^* algebra is the essential tool of axiomatic relativistic quantum field theory [1]. At the first step, let's show the construction of it.

Let B_i denote the sets contained in 4 dimensional Minkovski space. Let $\mathfrak{A}(B)$ denote C^* algebra related to B.

$\mathfrak{A}(B)$ has the following properties:

(1) To every relative compact open set B, one $\mathfrak{A}(B)$ is corresponded.

(2) If B_1 contains B_2, then $\mathfrak{A}(B_1)$ contains $\mathfrak{A}(B_2)$.

(3) If B_1 and B_2 are completely space like, then $\mathfrak{A}(B_1)$ and $\mathfrak{A}(B_2)$ are mutually commutative.

Let $\mathfrak{A}$ denote the completion of $\bigcup_B \mathfrak{A}(B)$ in quasi norm. $\mathfrak{A}$ is evidently C^* algebra.