83. An Aspect of Local Property of $|N, p_n|$ Summability of a Factored Fourier Series

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1. A series $\sum a_n$ with partial sums s_n is summable to sum s by the Nörlund method (N, p_n) if

$$t_n=\left\{\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k\right\}\to s, \tag{1.1}$$

as $n\to\infty$, where $P_n=\sum_{\nu=0}^{n} p_n$ and $p_\nu>0$ [2]. The series $\sum a_n$ is said to be absolutely summable (N, p_n), or summable $|N, p_n|$, if the sequence $\{t_n\}$ is of bounded variation [4]. The conditions for the regularity of the summability (N, p_n) defined by (1.1) are

$$\lim_{n\to\infty} p_n/P_n=0, \text{ and } \sum_{\nu=0}^{n}|p_\nu|=0(P_n). \tag{1.2}$$

In the special case in which

$$p_n=\binom{n+\alpha-1}{\alpha-1}=\frac{\Gamma(n+\alpha)}{\Gamma(n+1)\Gamma(\alpha)} \quad (\alpha>0),$$

the Nörlund mean reduces to the familiar Cesàro mean of order α [2]. And for the value for which

$$p_n=\frac{1}{n+1};\ P_n\sim\log n,$$

the Nörlund mean reduces to the harmonic mean [6].

Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without any loss of generality, we may assume that the constant term in the Fourier series of $f(t)$ is zero, that is,

$$\int_{-\pi}^{\pi} f(t)dt=0,$$

and

$$f(t)\sim\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=1}^{\infty}A_n(t).$$

We use the following notations:—

$$\phi(t)=\frac{1}{2}\left\{f(x+t)+f(x-t)-2f(x)\right\},$$

$$\Phi_\alpha(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-u)^{\alpha-1}\phi(u)du \quad (\alpha>0),$$

$$\Phi_0(t)=\phi(t),$$

$$\phi_\alpha(t)=\Gamma(\alpha+1)t^{-\alpha}\Phi_\alpha(t) \quad (0\leq\alpha\leq1).$$

2. In 1957 Prasad and Bhatt [5] established the following theorem: