132. *The Area of Nonparametric Measurable Surfaces*

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1. Basic notions. We shall understand by a *rectangle* any closed nondegenerate interval of the Euclidean plane $\boldsymbol{R}^2$. The letter I will be reserved to denote a rectangle. Let $I=[a_1, b_1; a_2, b_2]$ explicitly. When $0<\alpha<1$ and $2\alpha<\min(b_1-a_1, b_2-a_2)$, we say that α is *admissible* for I and we find it convenient to write

$$I_\alpha=[a_1+\alpha, b_1-\alpha; a_2+\alpha, b_2-\alpha].$$

Further, Rec I will denote the class of all subrectangles of I (inclusive of I itself).

Suppose that T is an *additive continuous map* of Rec I into the Euclidean space $\boldsymbol{R}^m$ of dimension m. In other words, let the m coordinates of the point $T(J)$, where $J\in\text{Rec}\, I$, be additive continuous functions of J in the usual sense [Saks 4, Chap. III]. If α is any admissible number for I, the quotient

$$T_\alpha(x, y)=T([x-\alpha, x+\alpha; y-\alpha, y+\alpha])/(4\alpha^2),$$

defined for the points $\langle x, y\rangle$ of the rectangle I_α, is obviously a continuous map of I_α into the space $\boldsymbol{R}^m$. We may say that T_α is the *squarewise mean* of T (for squares of side-length 2α).

Let g denote generically a continuous map of a rectangle K into $\boldsymbol{R}^m$, and let Ψ be a functional which assigns to each g a nonnegative value $\Psi(g)=\Psi(g; K)\leqq+\infty$. (It should be noted that not only the map g, but also the rectangle K is supposed arbitrary; the space $\boldsymbol{R}^m$, however, is kept fixed.) If J is a subrectangle of K, the partial map $g|J$ is continuous on J and we shall write $\Psi(g; J)$ for $\Psi(g|J)$.

Given as above the map T and the functional Ψ, let t be a generic continuous map of I into $\boldsymbol{R}^m$. We shall denote by $M(\Psi, T)$, or more expressly $M(\Psi, T; I)$, the lower limit of $\Psi(t; I_\alpha)$ as $\alpha\to 0$ and $\rho(T_\alpha, t; I_\alpha)\to 0$ simultaneously, where $\alpha, I_\alpha, T_\alpha$ have the aforesaid meaning and ρ indicates the ordinary distance, on I_α, between the two maps T_α and t. In other words, $M(\Psi, T)$ means the supremum of $M(\beta, \Psi, T)$ for all $\beta>0$, where $M(\beta, \Psi, T)$ is the infimum of $\Psi(t; I_\alpha)$ for all pairs $\langle\alpha, t\rangle$ such that $\alpha<\beta$ and $\rho(T_\alpha, t; I_\alpha)<\beta$. (The last inequality is fulfilled if, for example, we choose for t any continuous extension of T_α to the whole rectangle I.)

2. Aim of the note. By a *nonparametric measurable surface* we shall mean a surface of the form $z=f(x, y)$, where f is a finite measurable function on a rectangle. We are interested in the theory