152. *On the Perturbation of the Continuous Spectrum of the Dirac Operator*

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1. **Introduction.** In this note we are concerned with the Dirac operator arising in the relativistic quantum theory. Our purpose is to show the unitary equivalence between the free Dirac operator and the continuous part of the perturbed operator under certain conditions on the perturbation. The similar problems concerned with the Schroedinger operator have been studied by several authors. Among them the work of Friedrichs [3] is of interest from the viewpoint of time independent construction of the wave operator. Recently this so-called stationary method was developed by Faddief [2]. The method used here is essentially the same to that of Faddief.[1)]

2. **Operators in the momentum space.** Let $\mathcal{H}$ be the set of four-component vector-valued functions $f(\xi)\equiv(f_1(\xi),\cdots,f_4(\xi))$ defined on E_3 such that $f_k(\xi)\in L^2(E_3)$ $(k=1,2,3,4)$. $\mathcal{H}$ forms a Hilbert space with respect to the inner product

$$\langle f, g\rangle \equiv \int_{E_3} f(\xi)\cdot\overline{g(\xi)}d\xi = \int_{E_3}\sum_{k=1}^{4} f_k(\xi)\overline{g_k(\xi)}d\xi.$$

The free Dirac operator is given, as a multiplicative operator in $\mathcal{H}$, by

$$L_0 f \equiv L_0(\xi)f(\xi) = \left\{\sum_{k=1}^{3} A_k\xi_k + A_4\right\} f(\xi), \quad \xi=(\xi_1,\xi_2,\xi_3)\in E_3, \tag{2.1}$$

where A_k $(k=1,2,3,4)$ are constant matrices which satisfy the relations $A_jA_k+A_kA_j=2\delta_{jk}I$. It turns out that L_0 is a self-adjoint operator with the domain $\mathcal{D}\equiv\{f\in\mathcal{H};\ (1+|\xi|)f_k(\xi)\in L^2(E_3),\ k=1,2,3,4\}$. Moreover L_0 has no eigen-value and the continuous spectrum consists of all real λ such that $|\lambda|\geqslant 1$.

Let V be an integral operator

$$Vf = \int_{E_3} V(\xi-\eta)f(\eta)d\eta. \tag{2.2}$$

Throughout what follows we assume the matrix-valued function $V(\xi)\equiv[v_{jk}(\xi)]_{j,k=1,2,3,4}$ satisfies the following conditions.

(C.1) $v_{jk}(\xi)=\overline{v_{kj}(-\xi)}$; $\quad |v_{jk}(\xi)|\leqslant \text{const}\,(1+|\xi|)^{-2-\varepsilon_0}$;
$|v_{jk}(\xi)-v_{jk}(\xi+h)|\leqslant \text{const}\,(1+|\xi|)^{-2-\varepsilon_0}|h|^{\gamma_0}$,

1) Our problems are reduced to that of singular integrals with respect to the functions with Hoelder-continuity.