148. *Differential Forms of the Second Kind on Algebraic Varieties with Certain Imperfect Ground Fields**)

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1. Let V be an irreducible projective variety of dimension r defined over a field k of characteristic $p\neq 0$. We assume that the field K of rational functions of V over k is a regular extension of k. A subvariety W of V/k will mean a subvariety which is defined and irreducible over k; the local ring of W on V/k will be denoted by $\mathfrak{o}_W$, and the integral closure of $\mathfrak{o}_W$ in K by $\bar{\mathfrak{o}}_W$. A subvariety W of V/k is simple (over k) if and only if the local ring $\mathfrak{o}_W$ is regular.

We shall consider derivations and differential forms of K *over* k^p or, equivalently, over K^p. (They need not necessarily be trivial on k.) The exterior differential $d\omega$ of a differential form ω of K is therefore to be understood also in that sense. We have $d(z^p\omega)=z^pd\omega$, $z\in K$. Following the usual definition (in [3]) we define holomorphic differential forms and derivations as follows. A derivation ∂ of K is *holomorphic at a subvariety* W of V/k if $\partial\bar{\mathfrak{o}}_W\subseteqq\bar{\mathfrak{o}}_W$. A differential form ω of K of degree q is *holomorphic at* W if $\omega(\partial_1,\cdots,\partial_q)\in\bar{\mathfrak{o}}_W$ for all derivations $\partial_1,\cdots,\partial_q$ which are holomorphic at W. We denote by $\mathcal{D}_1(V)$ the k-vector space of all differential forms of K of degree 1 which are holomorphic at every subvariety of V/k. A differential form ω of K of degree q is *of the second kind at a subvariety* W if $\omega-d\theta_W$ is holomorphic at W with a suitable differential form θ_W of degree $q-1$. If ω is of the second kind at W, then $a^p\omega$ with $a\in k$ is also of the second kind at W. In fact, if $\omega-d\theta$ is holomorphic at W, then $a^p\omega-d(a^p\theta)=a^p(\omega-d\theta)$ is holomorphic at W since $a^p\in\bar{\mathfrak{o}}_W$. All *closed* differential forms of K of degree 1 which are of the second kind at every subvariety of V/k form therefore a k^p-vector space $\mathcal{D}_2(V)$, and the set $\mathcal{D}_e(V)$ of differentials dz $(z\in K)$ is a k^p-vector subspace of $\mathcal{D}_2(V)$. The purpose of the present note is to show that the dimension over k^p of the factor space $\mathcal{D}_2(V)/\mathcal{D}_e(V)$ equals the dimension over k of the space $\mathcal{D}_1(V)$ *under the assumption* $[k:k^p]<\infty$. In the case where k is perfect, our result implies the known equality $\dim_k\mathcal{D}_2(V)/\mathcal{D}_e(V)=\dim_k\mathcal{D}_1(V)$, which we have shown in [1]

*) After the completion of this paper, the auther found that the same result had been obtained in a more general form by E. Kunz in "Einige Anwendungen des Cartier-Operators", Arch. d. Math., **13**, 349–356 (1962). However, our treatment, using derivations and *uniformization* of relatively simple points introduced by Zariski-Falb [4], is essentially different from that of Kunz.