15. *Some Remarks on Von Neumann Algebras with an Algebraical Property*

By Hisashi CHODA and Marie ECHIGO
Department of Mathematics, Osaka Gakugei Daigaku
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Schwartz [8] established that there exists a pair of non-isomorphic, non-hyperfinite finite factors, so that he introduced the property P which is spatial one. In [2], we introduced a purely algebraical property, the property Q, and showed that the results of Schwartz were followed by the property Q. In [3], we proved that the crossed product $G\otimes\mathcal{A}$ of the finite von Neumann algebra $\mathcal{A}$ by a group G of outer automorphisms of $\mathcal{A}$ has the property Q only if G is amenable, and that the factor constructed by an enumerable ergodic m-group G on a measure space by the method due to Murray-von Neumann [4] is a continuous hyperfinite factor only if G is amenable. We obtained also in [3] a sufficient condition for the crossed product to have the property P.

In this paper, we shall report some further results on von Neumann algebras with the property Q. We shall publish the details in the Memoir of Osaka Gakugei University, Sect. B, No. 13 (1964).

In the below, we shall use the terminology of [3] without further explanations.

In the first place, a sufficient condition for the crossed product to have the property Q will be given in the following theorem:

THEOREM 1. *Let $\mathcal{A}$ be a von Neumann algebra with a finite faithful normal trace φ and G an amenable group of outer automorphisms of $\mathcal{A}$ such that*

$$\varphi(A^g)=\varphi(A) \qquad \text{for } g\in G \text{ and } A\in\mathcal{A}.$$

If $\mathcal{A}$ has an amenable generator $\mathcal{G}$ satisfying the following conditions:

i) $\mathcal{G}^g\subset\mathcal{G}$ *for any* $g\in G$,

and

ii) $\int f(U)\,dU^g=\int f(U)\,dU$ *for* $g\in G$ *and* $f\in L^\infty(\mathcal{G})$,

then the crossed product $G\otimes\mathcal{A}$ has the property Q.

Let Φ be the free group with two generators, then there exists a group G of outer automorphisms of the hyperfinite continuous factor $\mathcal{A}$ which is isomorphic to Φ, cf.[6]. For an infinite dimensional Hilbert space $\mathcal{K}$, let $\mathcal{B}=\mathcal{L}(\mathcal{K})$, then $(G\otimes\mathcal{A})\otimes\mathcal{B}$ is a factor of type II_∞ without the property Q. Hence, we have the following