14. A Remark on the Uniqueness of the Non-characteristic Cauchy Problem for Equations of Parabolic Type

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1. We shall consider the Cauchy problem of the equations written in the following form in $[-T, T]\times D$, where D is the closure of a domain with smooth boundary ∂D, in $n+1$-dimensional euclidean space $R^n_x \times R^1_t$;

$$(1)\qquad Pu=\Big(\frac{\partial^m}{\partial t^m}-\sum_{j=0}^{m-1}\sum_{j+m|\alpha:\mathfrak{m}|\leq m} a_{j,\alpha}(t, x)\frac{\partial^{j+|\alpha|}}{\partial t^j \partial x^\alpha}\Big)u(t, x)=0,$$

with the null initial data;

$$(2)\qquad \frac{\partial^\gamma}{\partial t^\gamma}u(0, x)=0 \quad x\in D,\ \gamma=0, 1, \cdots, m-1,$$

the notations contained in the above mean

m integer, $(t, x)=(t, x_1, \cdots, x_n)$

$\mathfrak{m}=(m_1, \cdots, m_n)$ m_j positive integers,

$\alpha=(\alpha_1, \cdots, \alpha_n)$ α_j non-negative integers and $|\alpha:\mathfrak{m}|=\sum_{j=1}^n \frac{\alpha_j}{m_j}$.

On this problem H. Kumanogo [1] and the author [2] obtained some results by the method of Carleman. But the both do not give any answer for the validity of the uniqueness in a neighborhood of the point where all $a_{j,\alpha}(t, x)$ vanish. On the other hand by eleving the regularity with respect to x and restricting the growth of derivatives of $a_{j,\alpha}(t, x)$, De Giorgi [3] obtained the uniqueness for (1) (2) in the case of two independent variables. We shall obtain an answer for the above question by extending De Giorgi's result for $n+1$ independent variables. The method is essentially the same as him. Recently G. Talenti [4] proved the uniqueness and existence for (1) with a special right hand side by extending M. Pucci's result [5] for two independent variables. His uniqueness theorem is for solutions in some Gevrey class and ours for genuine solutions. Y. Ôya [6] proved the existence and uniqueness of the Cauchy problem for the weakly hyperbolic equations which contain (1) as a special case and he assumes that $a_{j,\alpha}(t, x)$ are in some Gevery class with respect to both t and x, but with respect to t we only assume they are continuous.

2. *Theorem. We assume*

1) *There exist positive constants $A_{j,\alpha}$ and ρ such that*