12. Note on PL-Homeomorphisms of Euclidean n-Space into Itself

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(Comm. by Kinjirô KUNUGI, M.J.A., Jan. 12, 1965)

1. *Introduction.* Let $\mathcal{G}(n)$ be the space of all homeomorphisms of Euclidean n-space R^n into itself provided with the compact-open topology. Let $\mathcal{H}(n)$ be the subspace of all onto homeomorphisms. Let $Pl(n)$ be the subspace of all PL-homeomorphisms and $PL(n)$ be the subspace of all onto PL-homeomorphisms. Those elements in $\mathcal{G}(n)$, $\mathcal{H}(n)$, $Pl(n)$ and $PL(n)$ which preserve the origin 0 will be denoted by $\mathcal{G}_0(n)$, $\mathcal{H}_0(n)$, $Pl_0(n)$ and $PL_0(n)$ respectively. Recently Kister [1] has shown that $\mathcal{H}_0(n)$ is a weak kind of deformation retract of $\mathcal{G}_0(n)$.

In the present note we show that $PL_0(n)$ is a weak kind of deformation retract of $Pl_0(n)$. More precisely:

Theorem. *There is a continuous map F: $Pl_0(n)\times I\to Pl_0(n)$, for each n, such that*

(1) *$F(g, 0)=g$, for all g in $Pl_0(n)$,*
(2) *$F(g, 1)$ is in $PL_0(n)$ for all g in $Pl_0(n)$,*
(3) *$F(h, t)$ is in $PL_0(n)$ for all h in $PL_0(n)$, t in I.*

2. *Definitions.* Let R^n be a Euclidean n-space. We consider an ordinary triangulation on R^n. Let d be the usual metric in Euclidean n-space R^n. Let ρ be the metric in R^n defined by

$$\rho(x, y)=\max_i |x_i-y_i|,$$

for

$$x=(x_1, x_2, \cdots, x_n), \qquad y=(y_1, y_2, \cdots, y_n)$$

in R^n. The cube of side $2r$ with centre at 0 in R^n is denoted by C_r. This set is also considered as

$$C_r=\{x\in R^n \mid \rho(0, x)\leq r\}.$$

If K is a compact set in R^n containing 0, we define the *square radius* of K to be

$$r[K]=\max\{r \mid C_r\subset K\}.$$

If g_1, g_2: $K\to R^n$ are imbeddings of the compact set K, then we say g_1 and g_2 are *within* ε, if for each x in K it is true that $\rho(g_1(x), g_2(x))<\varepsilon$. If g is in $Pl_0(n)$ and K is a compact set in R^n, $V(g, K, \varepsilon)$ denotes the subset of all elements h in $Pl_0(n)$ such that $g|K$ and $h|K$ are within ε. Then the collection of all such $V(g, K, \varepsilon)$ is, of course, a base for $Pl_0(n)$.