11. *On the Sequence of Fourier Coefficients*

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1. Let $A:(d_{n,k})$, $n, k=0, 1, 2, \cdots$ and $d_{n,0}$, be a triangular Toeplitz matrix satisfying the conditions

(1.1) $$\lim_{n\to\infty} d_{n,k}=1 \quad \text{for every fixed } k,$$

and

(1.2) $$\sum_{k=0}^{n} |\Delta d_{n,k}| \leq K$$

where

$$\Delta d_{n,k}=d_{n,k}-d_{n,k+1}$$

and K being an absolute constant independent of n. It is easy to see that the third condition of Silverman Toeplitz theorem [page 64, 1] is automatically satisfied.

An infinite series $\sum u_n$ with the sequence of partial sum $\{S_n\}$ is said to be summable A to the sum S if

(1.3) $$\lim_{n\to\infty} \sum_{k=0}^{n} \Delta d_{n,k} S_k = S.$$

We obtain another method of summation viz. A. (C, 1) by superimposing the method A on the Cesàro mean of order one.

2. Let $f(x)$ be a function which is integrable in the sense of Lebesgue over the interval $(-\pi, \pi)$ and is defined outside this by periodicity. Let the Fourier Series of $f(x)$ be

(2.1) $$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n \cos nx+b_n \sin nx)=\frac{1}{2}a_0+\sum_{1}^{\infty}A_n(x),$$

then the conjugate series of (2.1) is

(2.2) $$\sum_{n=1}^{\infty}(b_n \cos nx-a_n \sin nx)=\sum_{1}^{\infty}B_n(x).$$

We write

$$\psi(t)=f(x+t)+f(x-t)-l.$$

Siddiqui [4] has proved that, if

(2.3) $$\sum_{k=0}^{n} |\Delta^2 d_{n,k}|=o(1)$$

and $\psi(t)$ is of bounded variation in $(0, \pi)$, then $\{nB_n(x)\}$ is summable A to l at $t=x$. Recently he [5] gave a necessary and sufficient condition on A for the validity of the above theorem.

The object of this paper is to prove the following theorem:

Theorem. If