7. *On Newman Algebras.* I

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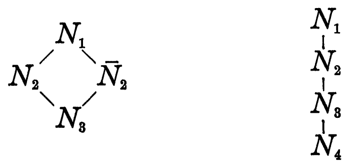
1. Introduction. The axiomatics of Newman algebras has been the subject of a number of papers by M. H. Newman [3], [4], G. D. Birkhoff and G. Birkhoff [1], [2], Y. Wooyenaka [7], [8], and F. M. Sioson [6]. In the present communication, a Newman algebra will be considered as an algebraic system $(N, {}^+, {}^\cdot, {}^-)$ with two binary operations $^+$ and $^\cdot$ and one unary operation $^-$.

For any postulate P of Newman algebras, let P^+ (similarly $P^\cdot$) denote the proposition obtained from P by commuting all the additions (multiplications) occurring in it. Thus, for instance, if P is $x(y\bar{y}+x)=x$, then $P^\cdot$ is $(\bar{y}y+x)x=x$. Note that, in general, $P^{++}=P=P^{\cdot\cdot}$, $P^{+\cdot}=P^{\cdot+}$, and if no $^+$ (no $^\cdot$) occurs in P, then $P^+=P(P^\cdot=P)$. Obviously, the propositional transformations $^+$ and $^\cdot$ thus defined generate an abelian group G_4 with four elements.

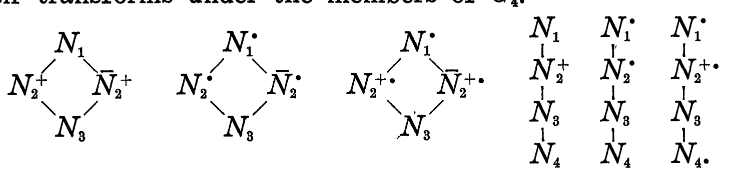
The author has previously shown in [6] that the only independent systems of postulates or *equational bases* for Newman algebras one can choose out of the following nine equations

$$N_1:\ x(y+z)=xy+xz,$$
$$N_2:\ x(y+\bar{y})=x, \qquad \bar{N}_2:\ x+y\bar{y}=x,$$
$$N_3:\ xy=yx, \qquad \bar{N}_3:\ x+y=y+x,$$
$$N_4:\ x(y\bar{y})=y\bar{y},$$
$$N_5:\ xx=x,$$
$$N_6:\ \bar{\bar{x}}=x,$$
$$\bar{N}_7:\ x+(y+z)=(x+y)+z,$$

are the systems:



and their transforms under the members of G_4:



In fact, it can be shown that these are all the equational bases for Newman algebras with the least possible number of equations (i.e.