3. *Extensions of Topologies*

By Shogo IKENAGA and Iwao YOSHIOKA

Department of Mathematics, Osaka Gakugei Daigaku

(Comm. by Kinjirô KUNUGI, M.J.A., Jan. 12, 1965)

Let (X, τ) be a topological space and $\tau \subset \tau^*$. Then τ^* will be called a simple extension of τ if and only if there exsists an $A \subset X$ such that $\tau^* = \{O \cup (O' \cap A) \mid 0, 0' \in \tau\}$. In this case we write $\tau^* = \tau(A)$. This definition is due to N. Levine [2]. N. Levine has obtained some interesting results about simple extensions of topologies [2].

It is the purpose of this note to consider the simple extensions of regular or other several topologies. In the next, we shall consider a generalization of simple extensions in § 3.

Let (X, τ) be a topological space and $\tau^* = \tau(A)$. Then we shall notice that for each $x \notin A$, the τ-open neighborhood system of x is a τ^*-open base of x and for each $x \in A$, the family $\{V(x) \cap A \mid V(x)$: τ-open neighborhood of $x\}$ is a τ^*-open base of x. Thus it is sufficient to consider these open bases.

The notations which will be used in this note are chiefly following. A^c denotes the complement of A. $\bar{A}$ and $\bar{A}^*$ denote the closure operators relative to τ and τ^* respectively. By $U(x)$, $V(x)$, and $W(x)$ we denote τ-open neighborhoods of x. $(A, \tau \cap A)$ denotes the subspace A of (X, τ), that is, $\tau \cap A$ denotes the relative topology of A with respect to τ.

The following facts have been shown in Lemma 3 of [2]. Let (X, τ) be a topological space and $\tau^* = \tau(A)$. Then $(A, \tau \cap A) = (A, \tau^* \cap A)$ and $(A^c, \tau \cap A^c) = (A^c, \tau^* \cap A^c)$. This follows from the above remark about the τ^*-open base of x.

§ 1. Simple extensions of regular topologies. In this section, we shall obtain a result about simple extensions of regular topologies which is better than N. Levine's theorem [2] and its application.

Let (X, τ) be a topological space and A a subset of X. We shall say that A is R-open in (X, τ) if and only if for each $x \in A$, there exists a $V(x)$ such that $V(x) \cap \bar{A} \subset A$, i.e., A is open in $(\bar{A}, \tau \cap \bar{A})$.

Theorem 1.1. *Let (X, τ) be a regular space and $\tau^* = \tau(A)$. Then the following conditions* (i)$\sim$(iii) *are equivalent*:

(i) *A is R-open in (X, τ);*

(ii) *$\bar{A} \cap A^c$ is closed in (X, τ);*

(iii) *(X, τ^*) is regular.*

Proof. It is evident that (i) and (ii) are equivalent. Then we