31. *Approximative Dimension of a Space of Analytic Functions*

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Banach [1] introduced the concept of linear dimension into the theory of topological linear spaces. Extending the idea, Kolmogorov [2] defined the *approximative dimension* with a view to more definite comparison between dimensions of certain linear spaces. Besides the definition he gave some of its examples in the note [2]. Among them we find a formula determining the approximative dimension for the space A_G^s of regular analytic functions defined on a domain G of s complex variables, with which the comparison of dimensions for different s leads to a reasonable result. The proof is not given, only it is mentioned that the formula can be derived by the same method as is used for the evaluation of ε-entropies. But A_G^s with the topology considered here being countably normed, i.e. not having such a simple metric as is usually taken to define ε-entropies, the circumstances are somewhat more complicated, the proof of the formula seems by no means trivial.

The purpose of the present paper is to give a complete proof to the formula in the simplest case where $s=1$ and $G=\{z: |z|<1\}$. In the proof we use some results in the theory of ε-entropies [3]. For general s, because those results are also available, the proof given here remains unchanged in essentials, as long as G is suitably simple so that it can be reduced to a polycylinder.

DEFINITION (Kolmogorov). To every topological linear space E we assign such a family $\Phi(E)$ of functions $\varphi(\varepsilon)$ defined for $\varepsilon>0$ as follows. A function $\varphi(\varepsilon)$ belongs to $\Phi(E)$ if and only if for every compact $K\subset E$ and every open neighborhood U of zero in E there exists a positive number ε_0 such that, when $\varepsilon<\varepsilon_0$, we can find $N\leqq\varphi(\varepsilon)$ points $x_1, \cdots, x_N$ in E forming a ε-net of K relative to U, i.e.

$$K\subset \bigcup_{i=1}^{N}(x_i+\varepsilon U).$$

The family $\Phi(E)$ is called the *approximative dimension* of E.[1]

Now let A_G be the space of regular functions on the open disk

1) In Kolmogorov's original definition, the approximative dimension $d_a(E)$ of E is not the family $\Phi(E)$ itself, but defined by comparison: for two topological linear spaces E and E', $d_a(E)\leqq d_a(E')$ if and only if $\Phi(E)\supset\Phi(E')$.