# 28. *Subsemigroups of Completely 0-Simple Semigroups. I*[*)]

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**1. Introduction.** A completely 0-simple semigroup $S$ is isomorphic to a regular Rees matrix semigroup over a group with zero $G^0=G\cup\{0\}$ with sandwich matrix $P=(p_{ji})$, $p_{ji}\in G^0$, $i\in L_0$, $j\in M_0$ where each row and each column of $P$ contains at least one non-zero element [1, 2, 3]. That is to say,

$$S=\{0\}\cup\{(x;\, i, j)\mid x\in G,\ i\in L_0,\ j\in M_0\}$$

where the multiplication is defined as follows:

$$0\cdot(x;\, i, j)=(x;\, i, j)\cdot 0=0\cdot 0=0 \qquad \text{for all } (x;\, i, j)$$

$$(x;\, i, j)\cdot(y;\, k, l)=\begin{cases}0 & \text{if } p_{jk}=0,\\ (xp_{jk}y;\, i, l) & \text{if } p_{jk}\neq 0.\end{cases}$$

$G$ is called the structure group of $S$.

It is known that any subsemigroup of a finite complete 0-simple semigroup $S$ is completely 0-simple if $P$ contains no zero [1, Ex. 19, p. 85]. This is not true for the general case without assumption of finiteness. Actually the type of subsemigroups of completely 0-simple semigroups is the generalization of completely 0-simple semigroups. The purpose of this series of the papers is to determine all subsemigroups of completely 0-simple semigroups. However, as the first step towards this study, the present paper treats 0-simple subsemigroups of completely 0-simple semigroups in the special case where $G^0$ is finite. In such a case, any subsemigroup of $S$ is completely 0-simple, or simple, if $P$ contains no zero; any 0-simple subsemigroup of $S$ is completely 0-simple if $P$ contains zero. Also we discuss how to construct such subsemigroups in a given $S$. We remark that the discussions in the case where $P$ contains no zero includes those in the case where $S$ is completely simple [1, 2] since, if $S$ is a completely simple semigroup and if $S^0$ denotes a completely 0-simple semigroup such that $S^0=S\cup\{0\}$, then any subsemigroup of $S^0$ containing 0 is a subsemigroup of $S$ with 0 adjoined.

The detailed proof will be published elsewhere.

**2. Support.** We start with subsemigroups of a completely 0-simple semigroup $R$ in which the structure group $G$ of $R$ is the

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