27. Harmonic Summability of a Sequence of Fourier Coefficients

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1. Let $f(t)$ be a function which is integrable in the Lebesgue sense over the interval $(-\pi, \pi)$ and is defined outside this interval by periodicity. Let the Fourier series of $f(t)$ at $t=x$ be

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n \cos nx+b_n \sin nx)=\sum_1^{\infty} A_n(x). \tag{1.1}$$

Then the conjugate series of (1.1) is

$$\sum_{n=1}^{\infty}(b_n \cos nx-a_n \sin nx)=\sum_1^{\infty} B_n(x). \tag{1.2}$$

We write

$$\psi(t)=f(x+t)-f(x-t)-l. \tag{1.3}$$

Definition: The series $\sum a_n$ with the sequence of partial sum $\{S_n\}$ is said to be summable of harmonic means or summable $(H, 1)$, if

$$\operatorname*{Lt}_{n\to\infty} \frac{1}{\log n}\sum_{k=0}^{\infty}\frac{S_{n-k}}{k+1}=s, \tag{1.4}$$

where s is a definite number. When condition (1.4) is satisfied with S_n replaced by $\{S'_n\}$ then the above series will be said to be summable $(H, 1)$ (C, 1), where S'_n denotes the n-th Cesàro mean of order one of the sequence $\{S_n\}$.

The object of this paper is to prove the following

THEOREM: If

$$\Psi(t)=\int_0^t|\psi(u)|\,du=o(t) \quad \text{as } t\to 0, \tag{1.5}$$

$$\int_{\pi/n}^{\delta}\frac{|\psi(u+\pi/n)-\psi(u)|}{u}\log\frac{1}{u}du=o(\log n), \tag{1.6}$$

then the sequence $\{nB_n(x)\}$ is summable $(H, 1)$ $(C, 1)$ to the value l/π.

This theorem generalizes the result of Varshney [4].

2. We require the following lemmas:

Lemma 1. [3] For all values of n and x,

$$\sum_{r=1}^{n}\frac{\sin rt}{r}=o(1).$$

Lemma 2. [1] If $0<t<\pi$, Then

$$\sum_{r=1}^{n}\frac{\cos rt}{r}=o\left(\log\frac{1}{t}\right).$$