26. The Relation between (N, p_n) and $(\bar{N}, p_n)$ Summability

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We suppose, throughout this note, that

$$p_n>0, \quad \sum_{n=0}^{\infty} p_n=\infty,$$
$$P_n=p_0+p_1+\cdots+p_n, \quad n=0, 1, \cdots.$$

The Nörlund transformation (N, p_n) is defined as transforming the sequence $\{s_n\}$ into the sequence $\{t_n\}$ by means of the equation

$$t_n=\frac{1}{P_n}\sum_{\nu=0}^{n} p_{n-\nu}s_\nu . \tag{1}$$

As is well known, this transformation is regular if

$$\lim_{n\to\infty}\frac{p_n}{P_n}=0. \tag{2}$$

See Hardy [1], p. 64.

The discontinuous Riesz transformation $(\bar{N}, p_n)$ is defined as transforming the sequence $\{s_n\}$ into the sequence $\{u_n\}$ by means of the equation

$$u_n=\frac{1}{P_n}\sum_{\nu=0}^{n} p_\nu s_\nu . \tag{3}$$

This transformation is regular (see Hardy [1], p. 57).

As is easily seen, the transformations (N, p_n) and $(\bar{N}, p_n)$ take symmetric forms, hence we can expect the close relation between them. We shall prove here the following

Theorem 1. *Suppose that*

$$\{p_n\} \text{ is non-increasing,} \tag{4}$$

and that

$$p_n\geq\sigma>0, \quad n=0, 1, \cdots. \tag{5}$$

*Then $(\bar{N}, p_n)$ implies**) (N, p_n).

Proof. From (3) we have

$$s_n=\frac{P_n u_n-P_{n-1}u_{n-1}}{p_n}, \quad n=0, 1, \cdots,$$

with $P_{-1}=u_{-1}=0$. Hence, from (1),

*) Given two summability methods A, B, we say that A *implies* B if any sequence summable A is summable B to the same sum.