51. Inversive Semigroups. III

By Miyuki YAMADA

Shimane University and Sacramento State College
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§1. **Introduction.** This paper is the continuation of the previous papers [8] and [9].

A semigroup G is said to be *regular* if it satisfies the following condition:

(C) For any element a of G, there exists an element x such that $axa=a$.

For example, inversive semigroups introduced by [8] are regular.

Now, consider the identity

(P) $x_1x_2x_3\cdots x_n=x_{p_1}x_{p_2}x_{p_3}\cdots x_{p_n}$,

where $(p_1, p_2, p_3, \cdots, p_n)$ is a non-trivial permutation of $(1, 2, 3, \cdots, n)$.

Such an identity is called a *permutation identity*. If (P) is valid for any elements $x_1, x_2, x_3, \cdots, x_n$ of a semigroup M, then we shall say that *M satisfies* the permutation identity (P). For example, *commutativity* $x_1x_2=x_2x_1$ and *normality* $x_1x_2x_3x_4=x_1x_3x_2x_4$ are clearly permutation identities. A semigroup satisfying commutativity $x_1x_2=x_2x_1$ is usually called a commutative semigroup. Similarly, we shall say that a semigroup is *normal* if it satisfies normality $x_1x_2x_3x_4=x_1x_3x_2x_4$. It is clear that any group satisfying a permutation identity is commutative. Further, we shall show later that any inverse semigroup introduced by Vagner [5] under the name "generalized group" is commutative if it satisfies a permutation identity. However, a regular semigroup satisfying a permutation identity is not necessarily commutative, and is sometimes quite different from commutative semigroups. This is easily seen from the fact that a rectangular band R is a regular semigroup satisfying normality, but any two different elements of R do not commute.[1] Special kinds of regular semigroups satisfying permutation identities have been studied by many papers (e.g., Clifford [1], Preston [3], Clifford & Preston [2], Thierrin [4], the author [6], [8], [9] and Kimura & the author [10]). Especially, Clifford [1] and the author [7] completely determined the structure of commutative inversive semigroups and gave an explicit description of a method of constructing all possible commutative inversive semigroups. On the other hand, Kimura & the author [10] clarified the structure of bands satisfying various

1) See Clifford and Preston [2], p. 26.