57. On Tannaka's Conjecture on the Cohomologically Trivial Modules

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Let G be a finite group. A G-module A is called cohomologically trivial when we have $H^q(H, A)=0$ for all integers q and all subgroups H of G. It is known that A is cohomologically trivial for the following cases:

(I) (Nakayama [1]) $H^r(H, A)=H^{r+1}(H, A)=0$ for some integer r and for all subgroups H of G.

(II) (Nakayama [1], [2]) G is a p-group, and $H^r(G, A)=H^{r+1}(G, A)=0$ for some integer r.

(III) (Nakayama [2]) G is a p-group, A is a G-module such that $pA=0$, and $H^r(G, A)=0$ for some integer r.

T. Tannaka proved that we could replace $r, r+1$ by $r, r+s$ in the first case, where s is a fixed odd integer (Th. 1). He also conjectured that the same would be true in the second case. Here we consider this problem in some special cases, while the general case is still open.

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1. We first recall the fundamental exact sequences. Let G be a finite group and H a normal subgroup of G. If A is a G-module, we denote by A^H the set of H-invariant elements, and by A_H the factor module of A by I_HA, where I_HA is a submodule of A generated by the elements of the form $(\sigma-1)a$ for σ in H and a in A. N_H denotes a norm map with respect to H, and ${}_{N_H}A$ denotes the kernel of N_H in A. We have then the following exact sequences

(1) $0\to H^1(G/H, A^H)\to H^1(G, A)\to H^1(H, A)^G\to H^2(G/H, A^H)\to H^2(G, A)$

(2) $0\to H^0(G/H, N_HA)\to H^0(G, A)\to H^0(H, A)^G\to H^1(G/H, N_HA)$
$\to H^1(G, A)$

(3) $0\to H^{-1}(G/H, A_H)\to H^{-1}(G, A)\to H^{-1}(H, A)^G\to H^0(G/H, A_H)$
$\to H^0(G, A)$

(4) $0\leftarrow H^0(G/H, A^H)\leftarrow H^0(G, A)\leftarrow H^0(H, A)_G\leftarrow H^{-1}(G/H, A^H)$
$\leftarrow H^{-1}(G, A)$

(5) $0\leftarrow H^{-1}(G/H, N_HA)\leftarrow H^{-1}(G, A)\leftarrow H^{-1}(H, A)_G\leftarrow H^{-2}(G/H, N_HA)$
$\leftarrow H^{-2}(G, A)$