86. Notes on (m, n)-Ideals. III

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The first two papers of this series are [2] and [3].

Let S be a semigroup. An (m, n)-ideal of S is called *locally minimal* if it contains no proper (m, n)-ideal. If a semigroup S contains no proper (m, n)-ideal, where m, n are arbitrary fixed positive integers, then by Theorem 4, S is a group. Thus we have the following result.

Theorem 10. *The locally minimal (m, n)-ideals of a semigroup S are groups. (m, n are arbitrary positive integers.)*

In case of $m=n=1$, Theorem 10 gives the

Corollary. *The locally minimal bi-ideals in a semigroup S are groups.*

An (m, n)-ideal A of a semigroup S is called *minimal*, if it does not properly contain any (m, n)-ideal of S. We prove the

Theorem 11. *Any locally minimal (m, n)-ideal of a semigroup S is also a minimal (m, n)-ideal of S.*

Proof. Let S be a semigroup, A a locally minimal (m, n)-ideal of S. If B would be an (m, n)-ideal of S, which is properly contained in A, then by Theorem 1, B would be an (m, n)-ideal of the semigroup A, because of $B=A\cap B$. But A has no proper (m, n)-ideal, thus A is indeed minimal (m, n)-ideal of S.

We shall call an (m, n)-ideal of a semigroup S *universally minimal* in S, if it is contained in every (m, n)-ideal of S. Obviously, the universally minimal (m, n)-ideal of S is also minimal. Such an universally minimal (m, n)-ideal of S is uniquely determined, as easy to see. Concerning universally minimal (m, n)-ideal of a semigroup S we prove the

Theorem 12. *Let S be a semigroup having a two-sided ideal G, which is at the same time a subgroup of S. Then G is the universally minimal (m, n)-ideal of S. (m, n are arbitrary non-negative integers.)*

Proof. Suppose that S is a semigroup having a two-sided ideal G, which is a subgroup of S. Then G is an (m, n)-ideal of S, for any non-negative integers m, n. Let A be an arbitrary (m, n)-ideal of S. Then

$$A^mGA^n\subseteq A^mSA^n\subseteq A.$$