

80. Standard Form in PGO and Transformation Algorithm: Problem-Solving Machines. II

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1. **Definitions.** In the former paper "Problem-Solving Machines, I", we reported on a method to produce the code expression from any natural language sentence of plane geometry, providing the code system PGO, and to retrieve the proof of any given theorem by machine. Now, we provide below some efficient standardization of any logical expression in plane geometry, named the *standard form*, and give a transformation algorithm.

Definition 1. A formula is a finite sequence of atomic formulas,¹⁾ logical symbols,²⁾ auxiliary symbols³⁾ except for comma.

The order of "binding" of logical symbols coincide with that of conventional mathematical usage, that is, in descending order by degree; \neg , \cap , \cup , \rightarrow .

Definition 2. An atomic formula is a literal; and if Q is an atomic formula then $\neg Q$ is a literal.

Definition 3. Well formed formula (wff): 1. An atomic formula is a wff. 2. If F is a wff, then $\neg F$ is a wff. 3-5. If E and F are wff's, then $E \cap F$, $E \cup F$, and $E \rightarrow F$ are wff's. 6. The only wff's are those given by 1-5.

2. **Standard form of the formula.** Theorems in plane geometry consist of the hypothesis and the conclusion part, that is, let A and B be wff's, the theorem is usually in the form

$$(2.1) \quad A \rightarrow B.$$

Using the disjunctive normal form in the hypothesis A and the conjunctive normal form in the conclusion B ,

$$(2.2) \quad \bigcup_{i=1}^{m_0} \bigcap_{j=1}^{n_i} A_{ij} \rightarrow \bigcap_{k=1}^{m_1} \bigcup_{l=1}^{n_k} B_{kl},$$

where A_{ij} and B_{kl} are literals.

It is easily seen that the disjunctions in the hypothesis and the conjunctions in the conclusion can be transferred to the front of the formula as conjunctions. Then we have

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1) atomic formula: A string of symbols consisting of a predicate letter followed by n terms.

2) logical symbols: \neg , \cap , \cup , \rightarrow , which mean negation, and, or, and imply.

3) auxiliary symbols: $(,)$ and comma.