114. *A Note on Elliptic Differential Operators of the Second Order*

By Mitio NAGUMO

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§ 0. **Introduction.** Let $x=(x_1, \cdots, x_m)$ donote a variable point in the euclidean m-space E_m and $\partial_i, \partial_{ij}$ denote partial differentiations $\partial/\partial x_i$ and $\partial^2/\partial x_i\partial x_j$ respectively. Let L be an elliptic differential operator of the second order with real coefficients:

$$Lu=\sum_{ij=1,1}^{m,m} a_{ij}(x)\partial_{ij}u+\sum_{i=1}^{m} b_i(x)\partial_i u$$

defined for $u\in C^2(G)$, where G is an open domain in E_m, $a_{ij}, b_i\in C(G)$ and the condition of ellipticity

$$A^{-1}|\xi|^2\leqq\sum_{ij=1,1}^{m,m} a_{ij}(x)\xi_i\xi_j\leqq A|\xi|^2$$

for all $\xi\in R^m$ and $x\in G$ holds with a fixed constant $A\geqq 1$.

K. Akô [1] gave an extension of the domain of the operator L as follows:

A function $u\in C(G)$ is said to be in the domain of $\mathcal{L}$ and satisfy the equation $\mathcal{L}u=f$ in G if the following conditions are satisfied

i) $f\in C(G)$.

ii) *For each* $x^0\in G$ *there exist a neighborhood* U *of* x^0 *and sequences*

$\{u_n\}_{n=1}^{\infty}\subset C^2(U)$, $\{a_{ij}^{(n)}\}_{n=1}^{\infty}\subset C(U)$, $\{b_i^{(n)}\}_{n=1}^{\infty}\subset C(U)$, *and* $\{f_n\}_{n=1}^{\infty}\subset C(U)$ *such that*

$$u_n\rightrightarrows u,\ a_{i,j}^{(n)}\rightrightarrows a_{ij},\ b_i^{(n)}\rightrightarrows b_i$$

and

$$L^{(n)}u_n=f_n\rightrightarrows f^{1)} \text{ in } U \text{ as } n\to\infty,$$

where

$$L^{(n)}=\sum_{i,j=1,1}^{m,m} a_{ij}^{(n)}(x)\partial_{ij}+\sum_{i=1}^{m} b_i^{(n)}\partial_i.$$

But as Akô has not proved the uniqueness of the extension, we shall here give the proof for the uniqueness.

In § 1 we shall consider the case where the sequence of the approximating operators $\{L^{(n)}\}$ is given previously independent of $\{u_n\}$, and give an elementary proof by Paraf's principle.

In § 2 we shall consider the general case from the viewpoint of the functional space (L^2). Then the proof will be at hand by virture of the recent theory of partial differential operators even for elliptic operators of arbitrarily high orders. We shall here, however, prefer more elementary way using a device of H. O. Cordes.

§ 1. **An elementary method.** In this section, for the sequence

1) The symbol $\rightrightarrows$ denotes the uniform convergence.