173. On Axiom Systems of Propositional Calculi. VIII

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In this note, we shall prove that the (R) axiom system 1∼6 of propositional calculus implies Sobosiński systems (S_1), (S_2) (for the notations and rules of inference, see [1]). In the fifth note [2] of this series, K. Iséki and S. Tanaka proved that Russell system implies Lukasiewicz (L_1)-system. And in the sixth note [3] it was proved that Lukasiewicz (L_2), (L_3)-systems, Hilbert system, and Mendelson system are deduced from the (R)-system.

1 $CpCqp$.

2 $CCpqCCqrCpr$.

3 $CCpCqrCqCpr$.

4 $CNNpp$.

5 $CCpNpNp$.

6 $CCpNqCqNp$.

3 r/p, $q/CpCqp$ *$C1$ $q/CpCqp$—$C1$—7,

7 Cpp.

6 p/Np, q/p *$C7$ p/Np—8,

8 $CpNNp$.

3 p/Cpq, q/Cqr, r/Cpr *$C2$—9,

9 $CCqrCCpqCpr$.

9 r/NNq *$C8$ p/q—10,

10 $CCpqCpNNq$.

2 p/Cpq, $q/CpNNq$, $r/CNqNp$ *$C10$—$C6$ q/Nq—11,

11 $CCpqCNqNp$.

9 q/NNq, r/q *$C4$ p/q—12,

12 $CCpNNqCpq$.

2 $p/CNpq$, $q/CNqNNp$, $r/CNqp$ *$C11$ p/Np—$C12$ p/Nq, q/p—13,

13 $CCNpqCNqp$.

9 $q/CNqp$, $r/CNpq$ *$C13$ p/q, q/p—$C1$ q/Nq—14,

14 $CpCNpq$.

3 q/Np, r/q *$C14$—15,

15 $CNpCpq$.

2 p/Cpq, $q/CCqrCpr$, r/s *$C2$—16,

16 $CCCCqrCprsCCpqs$.

16 q/Cqr, r/Csr, $s/CCsqCpCsr$ *$C16$ p/s, $s/CpCsr$—17,

17 $CCpCqrCCsqCpCsr$.