168. The Relation between (N, p_n) and $(\bar{N}, p_n)$ Summability. II

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§ 1. The present note is a continuation of the previous paper by the author [2]. We suppose, throughout this note,[1] that

$$p_n>0, \qquad \sum_{n=0}^{\infty} p_n=\infty,$$
$$P_n=p_0+p_1+ \cdots +p_n, \ n=0, 1, \cdots .$$

The Nörlund transformation (N, p_n) is defined as transforming the sequence $\{s_n\}$ into the sequence $\{t_n\}$ by means of the equation

$$t_n =\frac{1}{P_n} \sum_{\nu=0}^{n} p_{n-\nu}s_\nu. \tag{1}$$

As is well known, this transformation is regular if

$$\lim_{n\to\infty} \frac{p_n}{P_n}=0. \tag{2}$$

See Hardy [1], p. 64.

The discontinuous Riesz transformation $(\bar{N}, p_n)$ is defined as transforming the sequence $\{s_n\}$ into the sequence $\{u_n\}$ by means of the equation

$$u_n=\frac{1}{P_n} \sum_{\nu=0}^{n} p_\nu s_\nu. \tag{3}$$

This transformation is regular (see Hardy [1], p. 57).

From (1) we see easily

$$\sum_{\nu=0}^{n} P_{n-\nu}s_\nu= \sum_{\nu=0}^{n} P_\nu t_\nu.$$

Thus we obtain the following

Theorem 1. *(N, P_n) is equivalent*[2] *to the iteration product $(\bar{N}, P_n) \cdot (N, p_n)$.*

§ 2. We shall prove here the following

Theorem 2. *If*

$$\{p_n\} \text{ is non-increasing,} \tag{4}$$

and if

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1) In Lemma, we need not assume $\sum_{n=0}^{\infty} p_n = \infty$ generally.

2) Given two summability methods A, B, we say that A *implies* B if any series or sequence summable A is summable B to the same sum. We say that A and B are *equivalent* if A implies B and B implies A.