# 165. *On the Isomorphism Problem of Certain Semigroups Constructed from Indexed Groups*

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T. Tamura, in [1], has showed that the cancellative, archimedean, nonpotent, commutative semigroup $S$ can be constructed from the indexed group $G$ with index $I$, defining a product in $S=N_0\times G$, where $N_0$ is the set of all non-negative integers, by $(m, x)(n, y)=(m+n+I(x, y), xy)$ and proposed a problem that under what condition, is $S$ constructed from $G$ with $I$ isomorphic upon $S'$ from $G'$ with $I'$? In this paper, we shall give a solution without proofs for the above.

1. For any element $a$ of an indexed group $G$ with an index $I$ and any integers $r$ and $s$ we define $\rho_r^s(a)$ as follows:

$$\begin{aligned}\rho_r^s(a)&=\sum_{i=r}^{s} I(a, a^i) && \text{if } s-r\geqq 0,\\ &=0 && \text{if } s-r=-1,\\ &=-\sum_{i=s+1}^{r-1} I(a, a^i) && \text{if } s-r\leqq -2,\end{aligned}$$

where $a^0$ means the identity element of $G$.

Then we get the following lemmas:

Lemma 1. *For any integers $r$, $s$, and $t$ it holds that*

$$\rho_r^{s-1}(a)+\rho_s^t(a)=\rho_r^t(a).$$

Therefore, immediately

Lemma 2. *If the order of $a$ is $m$, then, for any integers $r$ and $s$*

$$\rho_1^{mr+s}(a)=r\rho_1^m(a)+\rho_1^s(a).$$

Lemma 3. *For any integers $r$ and $s$*

$$I(a^r, a^s)=\rho_s^{r+s-1}(a)-\rho_1^{r-1}(a)=\rho_r^{r+s-1}(a)-\rho_1^{s-1}(a).$$

From Lemmas 1 and 3

Lemma 4. *For any integer $r$*

$$\rho_1^r(a^{-1})=(r+1)\rho_{-1}^0(a)-\rho_{-r-1}^0(a).$$

2. Let $S=N_0\times G$ and $S'=N_0\times G'$ be cancellative, archimedean, nonpotent, commutative semigroups constructed from indexed groups $G$ with $I$ and $G'$ with $I'$ respectively. Suppose that $S$ is isomorphic upon $S'$ under $\varphi$. Let $e$ and $e'$ be the identity elements of $G$ and $G'$ respectively and put $(0, e)\varphi=(n', e_0')$ and $(0, e')\varphi^{-1}=(n, e_0)$. Since $(0, e')=(n, e_0)\varphi=((0, e_0)(0, e)^n)\varphi$, where we agree that $\alpha\beta^0$ means $\alpha$ for every $\alpha, \beta\in S$, we get

Lemma 5. $nn'=0$.